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On a Regular Field Theory.

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Summary. — A regular field theory is constructed in a finite space-time domain Ω restricted by two space-like hypersurfaces. The field equations are integral equations with regular kernels. The solutions are integer functions of the coupling parameter. Integral conservation laws are inferred. In order to secure the correspondence with the traditional field theory the form factor (introduced to regularize the kernels) must depend on the domain Ω. The integral field eq-s are quantized directly, without any reference to the canonical formalism. The Schrödinger-Tomonaga eq. does not exist but the formalism yields unambiguous results concerning the probability amplitudes connecting the measurements on the two hypersurfaces. The author represents the opinion that field quantization has nothing to do with the convergence difficulties. They disappear in the case of a finite domain and of regular kernels.

Introduction and summary.

Although the convergence difficulties inherent in the relativistic quantum field theory are known and extensively discussed for more than twenty years, their origin is not yet fully realized, and there is a discrepancy of opinions in this respect among physicists. Some of the authors believe that the reason of the difficulties is physical in character and, in order to get a consistent formalism, new physical assumptions are necessary (for example the introduction of a fundamental length λ). However, the introduction of a non-local point of view (necessarily connected with a fundamental length) was regarded, for many years, as impossible since the non-local fields resist quantization (in the traditional meaning of this word). This led some physicists to inves-

tigate the question of adequacy of the formalism of quantization which seems to be too tight to include more general types of fields and interactions. Among many attempts we note Heisenberg's S-matrix theory amounting to a refusal of the description of events in space-time.

Several authors represent an opposite viewpoint: they seem to believe that the usual quantum theory of localizable fields is consistent while the difficulties arise simply from the inadequacy of the traditional methods of solving approximatively the field equations (or solving the Schrödinger equation) by means of power series expansion in terms of the coupling constant. These authors (SCHWINGER, DYSON, FEYNMAN and others) try to improve the technique of the power expansion method by means of a procedure of renormalization of physical constants appearing in the field equations. They achieved a quantitative success in the electrodynamics but have not proved whether their procedures are unambiguous, and whether the series converges after the renormalizations. We note that the renormalization programm is not practicable in the case of most of meson theories.

From the mathematical viewpoint the possible reasons for the convergence difficulties encountered in the traditional field theories are threefold: (*i*) a singular (infinite) space time domain in which physicists used to investigate their field-theoretical problems, (*ii*) a singular character of the kernels of the integral field equations (which formally are equivalent to the traditional differential field equations), (*iii*) a singular character of the commutation relations between the field components. In most of the contemporary investigations these three possible reasons of difficulties appear hopelessly intermingled.

The first possible reason of difficulties may be removed simply by taking a well defined finite domain Ω where the system of fields undergoes an evolution starting with proper initial conditions.

The singular character of the kernels may be avoided by introducing a new physical assumption: a non-local interaction. This may be achieved by introducing a relativistic form factor discussed previously by the author [1] [2], and by KRISTENSEN and MÖLLER [3]. The existence and uniqueness of the solutions under the assumption of regular initial conditions is then easily demonstrated by means of the method due to PICARD.

Conservation laws applying to the surfaces enclosing the domain Ω are inferred. With a particular choice of the type of the form factor the formalism satisfies the principle of correspondence: the field equations and the expressions for the conserved quantities (charge, energy, momentum, and angular momentum) go over into the local ones in the limit $\lambda \to 0$.

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[1] J. RAYSKI: *Proc. Phys. Soc.*, **64**, 957 (1951).
[2] J. RAYSKI: *Phil. Mag.*, **42**, 1289 (1951).
[3] P. KRISTENSEN and C. M..LLER: *Det. Kgl. Danske Vid.* (in the press).

In this way it is shown that it is possible to avoid the first two kinds of singularities and to construct a satisfactory classical field theory whose field components are integer functions of the coupling constant. The solutions of the field equations may be computed by means of iteration.

The regular integral field equations may be quantized directly without a detour by the canonical formalism. Arguments are given in favour of the opinion that the quantization does not spoil the convergence in spite of the singular character of the commutation relations. The classical assumption restricting the initial values of the field components is to be replaced by a restriction upon the state vectors: only such state vectors should be admitted which yield limited expectation values of observables attached to the initial surface. We disagree with those authors who regard the general scheme of operators and state vectors as responsible for the difficulties encountered in the quantum theory. We believe that these difficulties are closely connected with the first two kinds of singularities, and thus are rather classical in character.

Although the general scheme of linear operators and state vectors with its associated probability interpretation remains unchanged, some of the aspects of the traditional local quantum field theory undergo profound modifications owing to the introduction of a non-local interaction. Above all we notice a preponderant role of the Heisenberg picture and the lack of a Schrödinger-Tomonaga equation. Then it is the action functional but not the Hamiltonian nor the S-matrix which is a primary concept. The S-matrix acquires a meaning merely as a limiting case of a more general matrix U_{12} describing the probability amplitudes for transitions between two space-like surfaces (enclosing the finite domain Ω) if this limit exists independently of the way in which the two surfaces are removed to plus and minus infinity. The transitions to the limit of an infinite domain and to the limit of the local interaction are briefly discussed and the necessity of further investigations of the problem of existence of solutions in these irregular cases is pointed out.

1. – The action functional.

Let us consider a set of field components (*) Φ^{α} with given transformation properties. α denotes, in short, any one of the sets of suffixes of the tensor or spinor field components. The field components are functions of the variables x_{μ} in a finite domain Ω of the four-dimensional continuum. We assume

(*) We follow the notations used in the cited papers of SCHWINGER but use natura units $\hbar = c = 1$.

that Ω is limited by two space-like hypersurfaces σ_1 and σ_2 of sufficient regularity and with a common edge B. We assume further that the set of fields forms an isolated system in Ω. The fields in Ω are characterized by an action functional W which is Lorentz invariant and invariant against transformations of the phase of the complex field variables by constant values (gauge invariance of the first kind). We assume further that the action functional may be split into two parts

$$W = W^{(0)} + W' , \tag{1.1}$$

where $W^{(0)}$ is the well known bilinear form of the field components Φ^x and their first derivatives Φ^x_μ which, in the absence of W', yields the Schrödinger-Gordon-Klein equations for the tensor components and the Dirac equations for the spinor components of the fields. W' describes the interaction between the fields. Let us consider the typical case where W' is formed of products of three field components, i.e. is a homogeneous bilinear form in the complex field functions and linear in the real field components. Let us assume also, for simplicity, that W' does not contain derivatives of the field components.

In order to build up a regular field theory we shall introduce a non localized interaction by taking the field components appearing in W' at different points x', x'', x''' respectively, and multiplying their product by a suitable form-function $F(x', x'', x'''; \Omega)$ which may depend upon the domain Ω.

$$W = W^{(0)} + W' = \int_\Omega \mathrm{d}x \mathcal{L}^{(0)}(x) + \iiint_\Omega \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \mathcal{L}'(x', x'', x'''; \Omega) . \tag{1.2}$$

In order to preserve the hermitian character of W the form function must satisfy the condition

$$F(x', x'', x'''; \Omega) = F^*(x''', x'', x'; \Omega) . \tag{1.3}$$

Of course, it must be invariant against the inhomogeneous Lorentz transformations.

Sometimes it will be convenient to illustrate the general arguments by a (purely academic) example of two coupled scalar fields: a complex ψ and a real φ. In the framework of the local field theory the simplest assumption for the interaction part of the action functional is

$$W' = \int \mathcal{L}'(x) \, \mathrm{d}x = g \int \mathrm{d}x \, \psi^*(x) \varphi(x) \psi(x) ,$$

where g is the coupling parameter. In the case of a non local interaction $\mathcal{L}'$

assumes the form

$$\mathcal{L}'(x', x'', x'''; \Omega) = gF(x', x'', x'''; \Omega)\psi^*(x')\varphi(x'')\psi(x'''). \tag{1.2'}$$

The variations of the field components by $\delta\Phi^\varkappa$ introduce a change $\delta_\Phi W$ of the action functional

$$\delta_\Phi W = \int_\Omega \mathrm{d}x \left(\frac{\partial \mathcal{L}^{(0)}(x)}{\partial \Phi^\varkappa_\mu(x)} \delta\Phi^\varkappa_\mu(x) + \frac{\partial \mathcal{L}^{(0)}(x)}{\partial \Phi^\varkappa(x)} \delta\Phi^\varkappa(x) \right) + \tag{1.4}$$

$$+ \iiint_\Omega \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, \frac{\partial \mathcal{L}'(x', x'', x'''; \Omega)}{\partial \Phi^\varkappa(x)} \delta\Phi^\varkappa(x) ,$$

where x in the last term denotes the one of the points x', x'', x''' which appears as argument of the function $\Phi^\varkappa$. An integration by parts yields

$$\delta_\Phi W = \int_\Omega \mathrm{d}x \left(- \frac{\partial}{\partial x_\mu} \frac{\partial \mathcal{L}^{(0)}}{\partial \Phi^\varkappa_\mu} + \frac{\partial \mathcal{L}^{(0)}}{\partial \Phi^\varkappa} + \iint_\Omega \mathrm{d}y \, \mathrm{d}z \, \frac{\partial \mathcal{L}'}{\partial \Phi^\varkappa} \right) \delta\Phi^\varkappa + \tag{1.5}$$

$$+ \left(\int_{\sigma_2} \mathrm{d}\sigma_\mu - \int_{\sigma_1} \mathrm{d}\sigma_\mu \right) \frac{\partial \mathcal{L}^{(0)}}{\partial \Phi^\varkappa_\mu} \delta\Phi^\varkappa ,$$

where y and z denote the other two of the points x', x'', x''' of the four dimensional continuum.

2. – Field equations.

The stationary action principle demands that the variation of the action functional should depend merely on the surface of the domain Ω. Thus, the variation $\delta_\Phi W$ should vanish if the variations $\delta\Phi^\varkappa$ vanish on fixed surfaces σ_1 and σ_2. This yields the Euler Lagrange equations

$$\frac{\partial}{\partial x_\mu} \frac{\partial \mathcal{L}^{(0)}(x)}{\partial \Phi^\varkappa_\mu(x)} - \frac{\partial \mathcal{L}^{(0)}(x)}{\partial \Phi^\varkappa(x)} = \iint_\Omega \mathrm{d}y \, \mathrm{d}z \, \frac{\partial \mathcal{L}'(x', x'', x'''; \Omega)}{\partial \Phi^\varkappa(x)} , \tag{2.1}$$

where x denotes again the one of the points x', x'', x''' which appears as argument of the function $\Phi^\varkappa$, and y, z denote the other two points.

We notice that the field equations (2.1) depend upon the domain Ω restricted by the two surfaces (where measurements take place). Hence, the

solutions must depend as well on Ω

$$\Phi^{\alpha} = \Phi^{\alpha}(x; \Omega) . \tag{2.2}$$

This is an essentially new feature of the field theory brought about by the non-local interaction.

The equations (2.1) are integro-differential equations. We replace them by pure integral equations. Let us introduce the Green function $G^{\sigma}(x, y)$ connected with the field Φ^{α} with the properties

$$\begin{cases} (\Box - m^2) G^{\sigma}_{\alpha}(x, y) = -\delta(x - y) , \\ G^{\sigma}_{\alpha}(x/\sigma, y) = 0 , \qquad n_{\mu}\partial_{\mu} G^{\sigma}_{\alpha}(x/\sigma, y) = 0 . \end{cases} \tag{2.3}$$

The symbol x/σ denotes a point x lying on the surface σ which is assumed to lie in Ω with the edge B common with the surfaces σ_1 and σ_2 n_{μ} is the unit vector normal to the surface σ. Denoting by σ_x another surface of the same type through the point x we have

$$G^{\sigma}_{\alpha}(x, y) = \begin{cases} -\Delta(x - y) & \text{for} \quad \sigma_x > y > \sigma \\ \Delta(x - y) & \text{»} \quad \sigma > y > \sigma_x \\ 0 & \text{elsewhere} , \end{cases} \tag{2.4}$$

where Δ is the well known Jordan Pauli function. (The symbol $>$ means « later » in this case).

(2.3) applies to the case of bosons. In the case of fermions the Green function is defined by

$$(\gamma_{\mu}\partial_{\mu} + m) G^{\sigma}(x, y) = -\delta(x - y) , \qquad G^{\sigma}(x/\sigma, y) = 0 , \tag{2.5}$$

and may be constructed explicitely with the aid of the well known function

$$S(x) = (\gamma_{\mu}\partial_{\mu} - m)\Delta(x) . \tag{2.6}$$

The Lagrange equations (2.1) are equivalent to the integral equations

$$\Phi^{\alpha}(x) = \Phi^{\alpha}_{o}(x) + \iiint_{\Omega} \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, G^{\sigma}_{\beta}(x, u) \frac{\partial \mathcal{L}'(x', x'', x'''; \Omega)}{\partial \Phi^{\beta}(u)} , \tag{2.7}$$

where $\Phi^{\alpha}_{\sigma}(x)$ denotes a solution of the interaction-free Schrödinger-Gordon equation (or of the Dirac equation), Φ^{β} denotes the complex conjugate of Φ^{α} (or the complex conjugate multiplied by γ_{μ} to the right in the case of spinors). u denotes this one of the points x', x'', x''' which appears as argument of the function Φ^{β}. From the properties (2.3) or (2.5) of the Green functions it is seen that Φ^{α} coincides with the free wave Φ^{α}_{σ} (together with the normal derivative in the case of bosons) on the surface σ.

We introduce the notation

$$K^{\sigma}_{\alpha}(x, y, z; \Omega) = \int_{\Omega} \mathrm{d}u\, G^{\sigma}_{\alpha}(x, u) F(x', x'', x'''; \Omega)\,, \tag{2.8}$$

where the integration variable u denotes the one of the points x', x'', x''' that appears as argument of the function Φ^{α} while y, z denote the remaining two points. In view of the properties of the Green functions we see that K^{σ}_{α} vanishes (together with its normal derivative in the case of bosons) for points x on the surface σ and satisfies the following equation

$$(\Box - m^2)K^{\sigma} = -F \quad \text{or} \quad (\gamma_{\mu}\partial_{\mu} + m)K^{\sigma} = -F\,, \tag{2.9}$$

in the cases of bosons or fermions respectively. The integral equations (2.7) may be written in terms of the kernels (2.8).

In the case of the two scalar fields the equations (2.1) are

$$\left\{\begin{aligned}
(\Box - m^2)\psi(x) &= -g\iint_{\Omega} \mathrm{d}x''\,\mathrm{d}x'''\, F(x, x'', x'''; \Omega)\varphi(x'')\psi(x''')\,,\\
(\Box - \mu^2)\varphi(x) &= -g\iint_{\Omega} \mathrm{d}x'\,\mathrm{d}x'''\, F(x', x, x'''; \Omega)\psi^*(x')\psi(x''')\,,\\
(\Box - m^2)\psi^*(x) &= -g\iint_{\Omega} \mathrm{d}x'\,\mathrm{d}x''\, F(x', x'', x; \Omega)\psi^*(x')\varphi(x'')\,,
\end{aligned}\right. \tag{2.10}$$

while the corresponding integral equations (2.7) are

$$\left\{\begin{aligned}
\psi(x) &= \psi_{\sigma}(x) + g\iiint_{\Omega} \mathrm{d}x'\,\mathrm{d}x''\,\mathrm{d}x'''\, G^{\sigma}_{\psi^*}(x, x')F(x', x'', x'''; \Omega)\varphi(x'')\psi(x''')\,,\\
\varphi(x) &= \varphi_{\sigma}(x) + g\iiint_{\Omega} \mathrm{d}x'\,\mathrm{d}x''\,\mathrm{d}x'''\, G^{\sigma}_{\varphi}(x, x'')F(x', x'', x'''; \Omega)\psi^*(x')\psi(x''')\,,\\
\psi^*(x) &= \psi^*_{\sigma}(x) + g\iiint_{\Omega} \mathrm{d}x'\,\mathrm{d}x''\,\mathrm{d}x'''\, G^{\sigma}_{\psi}(x, x''')F(x', x'', x'''; \Omega)\psi^*(x')\varphi(x'')\,,
\end{aligned}\right. \tag{2.11}$$

where G^{σ}_{ψ} and G^{σ}_{φ} are given by (2.3) with the mass parameter m or μ in the Jordan-Pauli functions. With the aid of (2.8) the equations (2.11) may be written in a more concise form

$$
(2.12)\qquad \begin{cases} \psi(x) = \psi_{\sigma}(x) + g \iint\limits_{\Omega} \mathrm{d}y\,\mathrm{d}z\, K^{\sigma}_{\psi^*}(x, y, z)\varphi(y)\psi(z)\,, \\ \varphi(x) = \varphi_{\sigma}(x) + g \iint\limits_{\Omega} \mathrm{d}y\,\mathrm{d}z\, K^{\sigma}_{\varphi}(x, y, z)\psi^*(y)\psi(z)\,, \\ \psi^*(x) = \psi^*_{\sigma}(x) + g \iint\limits_{\Omega} \mathrm{d}y\,\mathrm{d}z\, K^{\sigma}_{\psi}(x, y, z)\psi^*(y)\varphi(z)\,, \end{cases}
$$

where K^{σ}_{ψ} and K^{σ}_{φ} vanish, together with their normal derivatives, on σ and satisfy the equations

$$
(2.13)\qquad \begin{cases} (\Box - m^2)K^{\sigma}_{\psi^*}(x, x'', x''') = -F(x\,, x''\,, x'''; \Omega)\,, \\ (\Box - \mu^2)K^{\sigma}_{\varphi}(x, x', x''') = -F(x'\,, x\,, x'''; \Omega)\,, \\ (\Box - m^2)K^{\sigma}_{\psi}(x, x', x'') = -F(x'\,, x''\,, x\;; \Omega)\,. \end{cases}
$$

For the integral field equations ((2.7) or (2.11) or (2.12)) the following theorem holds:

Theorem: If the form-function F is suitably chosen, and if the initial values of the field functions on σ are limited, then the integral field equations possess a unique solution in the domain Ω and the series of approximations obtained by iteration converges to this solution for sufficiently small values of the coupling parameter.

A proof is given in the Appendix.

3. – Conservation laws and correspondence.

The conservation laws follow from the invariance of the action functional W with respect either to infinitesimal phase transformations or to infinitesimal rigid displacements and rotations of the domain Ω. The general variation of the action functional is

$$
(3.1)\qquad \delta W = \delta_{\Phi} W + \delta_{\sigma} W\,,
$$

where $\delta_{\Phi} W$ is the variation produced by the variations of the field components $\delta_0 \Phi^{\varkappa}$ at fixed points, and $\delta_{\sigma} W$ is the variation produced by the infinitesimal

variations of the boundaries σ_1 and σ_2. Taking account of the Lagrange equations we have from (1.5)

$$\delta_\Phi W = \delta_\Phi W^{(0)} = \left(\int_{\sigma_2} - \int_{\sigma_1} \right) \mathrm{d}\sigma_\mu \frac{\partial \mathcal{L}^{(0)}}{\partial \Phi^\varkappa_\mu} \delta_0 \Phi \,. \tag{3.2}$$

We notice that the variations of the field components effect only the interaction-free part of the action functional which is identical with that in the local field theory. Therefore we obtain the same expression for the charge as in the traditional theory: Performing infinitesimal variations of the phase of the complex field components we find as usually

$$\sum_\varkappa \varepsilon^\varkappa \int_{\sigma_2} \mathrm{d}\sigma_\mu \frac{\partial \mathcal{L}^{(0)}}{\partial \Phi^\varkappa_\mu} \Phi^\varkappa = \sum_\varkappa \varepsilon^\varkappa \int_{\sigma_1} \mathrm{d}\sigma_\mu \frac{\partial \mathcal{L}^{(0)}}{\partial \Phi^\varkappa_\mu} \Phi^\varkappa \,, \tag{3.3}$$

where $\varepsilon^\varkappa = \pm 1$, or 0 for complex field components, their complex conjugates, and the real field components respectively. Thus, apart from a constant factor, we may call the conserved quantity

$$Q = \int_\sigma \mathrm{d}\sigma_\mu \sum_\varkappa \varepsilon^\varkappa \frac{\partial \mathcal{L}^{(0)}}{\partial \Phi^\varkappa_\mu} \Phi^\varkappa \,, \tag{3.4}$$

the charge of the system, and

$$j_\mu = \sum_\varkappa \varepsilon^\varkappa \frac{\partial \mathcal{L}^{(0)}(x)}{\partial \Phi^\varkappa_\mu(x)} \Phi^\varkappa(x) \,, \tag{3.5}$$

the charge and current density fourvector. The charge is conserved on the boundaries σ_1 and σ_2 of the domain Ω. The analogy with the local field theory is however merely formal owing to the fact, that the Lagrange equations and their solutions depend upon the domain of integration. This domain may be taken arbitrarily small so that we may write formally $\lim_{\Omega \to 0} \partial_\mu j_\mu = 0$ but this « continuity equation » is an empty statement, since the whole field formalism becomes meaningless in the limit $\Omega \to 0$.

In order to infer the energy-momentum and the angular momentum conservation laws we carry out an infinitesimal rigid displacement and rotation of the domain Ω by

$$\delta x_\mu = \varepsilon_\mu - \varepsilon_{\mu\nu} x_\nu \,, \tag{3.6}$$

where ε_μ, $\varepsilon_{\mu\nu}$ are small arbitrary quantities. The change of the action functional under the displacement (3.6) is

$$\delta_\sigma W = \delta_\sigma W^{(0)} + \delta_\sigma W' , \tag{3.7}$$

where

$$\delta_\sigma W^{(0)} = \left(\int_{\sigma_1 + \delta\sigma}^{\sigma_2 + \delta\sigma} - \int_{\sigma_1}^{\sigma_2} \right) \mathrm{d}x \mathcal{L}^{(0)}(x) = \oint \mathrm{d}\sigma_\mu \, \delta x_\mu \mathcal{L}^{(0)}(x) , \tag{3.7'}$$

$$\delta_\sigma W' = \iiint_{\sigma_1 + \delta\sigma}^{\sigma_2 + \delta\sigma} \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, \mathcal{L}'(x', x'', x''' ; \Omega + \delta\Omega) - \tag{3.7''}$$

$$- \iiint_{\sigma_1}^{\sigma_2} \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, \mathcal{L}'(x', x'', x''' ; \Omega) .$$

At the same time we perform the variations $\delta_0 \Phi^\alpha$ in such a way that the new field components assume the same values on the surfaces $\sigma_2 + \delta\sigma_2$ and $\sigma_1 + \delta\sigma_1$ as the old did on the surfaces σ_2 and σ_1. The action functional is invariant against translations and rotations of the boundaries together with the field components whence the whole variation vanishes

$$\delta W = \delta W^{(0)} + \delta W' = \delta_\Phi W^{(0)} + \delta_\sigma W^{(0)} + \delta_\sigma W' = 0 , \tag{3.8}$$

with $\delta_\Phi W^{(0)}$ given by (3.2).

By taking ε_μ arbitrary and $\varepsilon_{\mu\nu} = 0$ we infer from (3.8) a tensor

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + T'_{\mu\nu} , \tag{3.9}$$

with the property

$$P_\nu(\sigma_1) = \int_{\sigma_1} \mathrm{d}\sigma_\mu T_{\mu\nu} = \int_{\sigma_2} \mathrm{d}\sigma_\mu T_{\mu\nu} = P_\nu(\sigma_2) , \tag{3.10}$$

so that P_μ may be interpreted as the energy and momentum fourvector. In the same way, by taking $\varepsilon_{\mu\nu}$ arbitrary and $\varepsilon_\mu = 0$, we find a tensor

$$M_{\lambda\mu\nu} = M^{(0)}_{\lambda\mu\nu} + M'_{\lambda\mu\nu} , \tag{3.11}$$

with the property

$$I_{\mu\nu}(\sigma_1) = \int_{\sigma_1} \mathrm{d}\sigma_\lambda \, M_{\lambda\mu\nu} = \int_{\sigma_2} \mathrm{d}\sigma_\lambda \, M_{\lambda\mu\nu} = I_{\mu\nu}(\sigma_2), \tag{3.12}$$

so that $I_{\mu\nu}$ may be interpreted as the angular momentum of the system.

We notice that the variations $\delta_\Phi W^{(0)}$ and $\delta_\sigma W^{(0)}$ are (formally) identical with those of the local field theory so that we may refer the reader to the literature ([4,5]) for details of the construction of the interaction-free part of the energy-momentum and angular momentum tensors $T^0_{\mu\nu}$ and $M^{(0)}_{\lambda\mu\nu}$. We shall limit ourselves to the construction of the interaction parts $T'_{\mu\nu}$ and $M'_{\lambda\mu\nu}$ which follow from the discussion of (3.7″). To compute these quantities we must know the dependence of $\mathcal{L}'$ on the domain Ω, that is, the dependence of the form-factor $F(x', x'', x'''; \Omega)$ on Ω. This dependence may be fixed unambiguously by invoking the requirement of correspondence with the local field theory.

In order the secure the correspondence let us introduce a parameter λ (with dimension of a length) into the form-factor F in such a way that the form-factor goes over into a product of two Dirac delta functions in the limit $\lambda \to 0$.

$$\lim_{\lambda \to 0} F(x', x'', x'''; \Omega) = \delta(x' - x'') \, \delta(x'' - x''') \, . \tag{3.13}$$

The form-factor must be invariant against displacements of the origin of the coordinate system, and therefore it is natural to introduce first a Lorentz invariant distribution function $R(x, y, z; \lambda)$ with the properties

$$R(x, y, z; \lambda) = R^*(z, y, x; \lambda) \, , \qquad \lim_{\lambda \to 0} R(x, y, z; \lambda) = \delta(x) \, \delta(y) \, \delta(z) \, . \tag{3.14}$$

With the aid of this distribution function (which should be regarded as primary) we may define the form-factor F either by

$$F_1(x', x'', x''') = \int_{\infty} \mathrm{d}x \, R(x' - x, x'' - x, x''' - x; \lambda) \, , \tag{3.15'}$$

which is independent of Ω, or by

$$F_2(x', x'', x'''; \Omega) = \int_{\Omega} \mathrm{d}x \, R(x' - x, x'' - x, x''' - x; \lambda) \, , \tag{3.15''}$$

([4]) W. Pauli: *Rev. Mod. Phys.* **13**, 203 (1941).
([5]) J. Schwinger: *Phys. Rev.*, **82**, 914 (1951).

which is referred to the domain Ω. More generally, we may consider a linear combination

$$F(x', x'', x'''; \Omega) = a_1 F_1 + a_2 F_2 . \tag{3.16}$$

It is easily seen that the Lagrange equations go over into the local ones if the coefficients of (3.16) satisfy the condition

$$a_1 + a_2 = 1 . \tag{3.17}$$

We shall show that the requirement of correspondence of the energy-momentum and of the angular momentum tensors with the local ones yields another condition for the coefficients of (3.16), namely

$$3a_1 + 4a_2 = 1 , \tag{3.18}$$

so that a_1 and a_2 are fixed unambiguously

$$a_1 = 3 , \qquad a_2 = -2 , \tag{3.19}$$

and we have no other choice for the form-factor F as regards its dependence on Ω. Thus, the two requirements of correspondence (of field equations and of the conserved quantities) are independent. In order to satisfy both requirements a simple form-factor (3.15′) or (3.15″) is not sufficient but we have to take their linear combination. In the limit where Ω becomes the whole space-time manifold F_1 and F_2 become identical and the formalism simplifies considerably.

In order to determine $T'_{\mu\nu}$ and $M'_{\lambda\mu\nu}$ we compute (3.7″)

$$\begin{aligned} \delta_\sigma W' = {} & \oint \mathrm{d}\sigma'_\mu \, \delta x'_\mu \iint_{\Omega} \mathrm{d}x'' \, \mathrm{d}x''' \, \mathcal{L}'(x' \, x'', x'''; \Omega) + \\ & + \oint \mathrm{d}\sigma''_\mu \, \delta x''_\mu \iint_{\Omega} \mathrm{d}x' \, \mathrm{d}x''' \, \mathcal{L}'(x', x'', x'''; \Omega) + \\ & + \oint \mathrm{d}\sigma'''_\mu \, \delta x'''_\mu \iint_{\Omega} \mathrm{d}x' \, \mathrm{d}x'' \, \mathcal{L}'(x', x'', x'''; \Omega) + \\ & + \iiint_{\Omega} \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, \delta_\sigma \mathcal{L}'(x', x'', x'''; \Omega) . \end{aligned} \tag{3.20}$$

In order to avoid unessential complications we restrict ourselves again to the simple case of two coupled scalar fields where $\mathcal{L}'$ is given by (1.2′) with F

given by (3.16). The last term in (3.20) becomes

$$g a_2 \oint \mathrm{d}\sigma_\mu \, \delta x_\mu \iiint_\Omega \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, R(x' - x, x'' - x, x''' - x) \psi^*(x') \varphi(x'') \psi(x''') \, ,$$

so that $\delta_\sigma W'$ may be written as

$$\begin{aligned} (3.21) \quad \delta W' = \oint \mathrm{d}\sigma_\mu \, \delta x_\mu \Big\{ & \iint_\Omega \mathrm{d}x'' \, \mathrm{d}x''' \, \mathfrak{L}'(x, x'', x'''; \Omega) + \\ & + \iint_\Omega \mathrm{d}x' \, \mathrm{d}x''' \, \mathfrak{L}'(x', x, x'''; \Omega) + \iint_\Omega \mathrm{d}x' \, \mathrm{d}x'' \, \mathfrak{L}'(x', x'', x; \Omega) \\ & + a_2 g \iiint_\Omega \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, R(x' - x, x'' - x, x''' - x) \psi^*(x') \varphi(x'') \psi(x''') \Big\} . \end{aligned}$$

If we consider a variation with $\varepsilon_{\mu\nu} = 0$ and ε_μ arbitrary then we may write $\oint \mathrm{d}\sigma_\mu \, \delta x_\mu \ldots \quad \oint \mathrm{d}\sigma_\mu \varepsilon_\nu \delta_{\mu\nu} \ldots$ whence the interaction part of the energy-momentum tensor follows

$$\begin{aligned} (3.22) \quad T'_{\mu\nu}(x) = g \delta_{\mu\nu} \Big\{ & \iint_\Omega \mathrm{d}x'' \, \mathrm{d}x''' \, F(x, x'', x'''; \Omega) \psi^*(x) \varphi(x'') \psi(x''') \\ & + \iint_\Omega \mathrm{d}x' \, \mathrm{d}x''' \, F(x', x, x'''; \Omega) \psi^*(x') \varphi(x) \psi(x''') + \\ & + \iint_\Omega \mathrm{d}x' \, \mathrm{d}x'' \, F(x', x'', x; \Omega) \psi^*(x') \varphi(x'') \psi(x) + \\ & + a_2 \iiint_\Omega \mathrm{d}x' \, \mathrm{d}x'' \, \mathrm{d}x''' \, R(x' - x, x'' - x, x''' - x) \psi^*(x') \varphi(x'') \psi(x''') \Big\} \end{aligned}$$

where

$$\begin{aligned} (3'22') \quad F(x, x'', x'''; \Omega) = a_1 \int_\infty \mathrm{d}x' \, R(x - x', x'' - x', x''' - x') + \\ + a_2 \int_\Omega \mathrm{d}x' \, R(x - x', x'' - x', x''' - x') \, , \end{aligned}$$

and so on.

By taking $\varepsilon_\mu = 0$ and $\varepsilon_{\mu\nu}$ arbitrary we find the angular momentum density tensor $M'_{\lambda\mu\nu}$ connected with $T'_{\mu\nu}$ in the usual way

$$(3.23) \qquad M'_{\lambda\mu\nu}(x) = x_\mu T'_{\lambda\nu}(x) - x_\nu T'_{\lambda\mu}(x) \, ,$$

so that the same formula (3.23) holds also for the complete tensor $M_{\lambda\mu\nu}(x) = M^{(0)}_{\lambda\mu\nu}(x) + M'_{\lambda\mu\nu}$. The interaction part of the energy-momentum density tensor is symmetrical. The interaction-free part may be symmetrized in the usual way so that the whole $T_{\mu\nu} = T^{(0)}_{\mu\nu} + T'_{\mu\nu}$ may be put into a symmetrical form as usually.

We see that the expression (3.22) is rather complicated since it contains seven terms altogether, three of which appear multiplied by a_1 and four by a_2. In the limit of localizability ($\lambda \to 0$) each of those seven terms goes over into the usual expression for $T'_{\mu\nu}$ of the local theory. Thus, we find

$$\lim_{\lambda\to 0} T'_{\mu\nu}(x) = (3a_1 + 4a_2) g \delta_{\mu\nu} \psi^*(x)\varphi(x)\psi(x) \tag{3.24}$$

wherefrom the condition (3.18) follows.

Going over to the limit of the whole space-time as the domain Ω F_2 becomes identical whit F_1 so that the form-factor as well as the Lagrange equations and the expressions for the density tensors are simplified considerably. For example, the interaction part $T'_{\mu\nu}$ becomes

$$\begin{aligned}\lim_{\Omega\to\infty} T'_{\mu\nu}(x) = g\delta_{\mu\nu} \iiint \mathrm{d}x' \,\mathrm{d}x'' \,\mathrm{d}x''' R(x - x', x'' - x', x''' - x'; \lambda)\psi^*(x)\varphi(x'')\psi(x''') + \\ + R(x' - x'', x - x'', x''' - x''; \lambda)\psi^*(x')\varphi(x)\psi(x''') + \\ + R(x' - x''', x'' - x''', x - x'''; \lambda)\psi^*(x')\varphi(x'')\psi(x) - \\ - 2R(x' - x, x'' - x, x''' - x; \lambda)\psi^*(x')\varphi(x'')\psi(x''').\end{aligned} \tag{3.25}$$

The results of this section may be summarized as follows. The regular field theory satisfies the conservation laws of quantities which may be interpreted as charge, energy and momentum, and angular momentum. The conservation laws refer to and only to the surface σ_1 and σ_2 enclosing the domain Ω. The two requirements (*i*) that the field equations go over into the local ones, and (*ii*) that the conserved quantities become identical with the usual ones in the limit $\lambda \to 0$, where λ is a fundamental length, are independent from each other. Only a special form of the form-factor (3.16) with F_1 and F_2 given by (3.15) and a_1 and a_2 given by (3.19) secures the correspondence with the local theory. The form-factor (3.16), the Lagrange equations (2.1) and the expression for the energy-momentum density tensor simplify considerably in the limiting case of the surfaces σ_1 and σ_2 tending to minus and plus infinity.

4. – Quantization and Interpretation.

We notice that some of the physical quantities attached to the surface σ_1 (or σ_2), as for example the charge, depend only upon the values of the field components on this surface while others, as for example the energy, involve integrals of the field components over the whole domain Ω. We call them localizable and non-localizable quantities respectively. The charge on a surface σ_1 (or σ_2) is a localizable quantity, the energy, though referring to the surface σ_1 (or σ_2) is a non-localizable quantity. The momentum and the space-space components of the angular momentum are localizable only if the surfaces to which they are referred are $t = \text{const}$. It is clear that only the quantities of the first kind may serve to determine the initial state of the system on a given surface.

The direct dependence of some of the physical quantities on the domain Ω as well as the dependence of the field equations on the domain Ω seems, at the first sight, unphysical. This new feature of the field theory becomes understandable if we agree to regard the surfaces σ_1 and σ_2 not as mere abstractions but as surfaces physically distinguished by the facts that on σ_1 the initial state has been prepared and on σ_2 the resulting state has been measured. Between these two surfaces the system is left to itself i.e. is isolated from interactions with the measuring apparatus. From the point of view of this interpretation it seems quite natural that conservation laws are stated with respect to the surfaces σ_1 and σ_2 where measurements actually take place. A field formalism with a non-localized interaction introduces a fundamental length λ into the theory. The incorporation of new (dimensional) constants has been always connected with new problems within the theory of measurements. Thus, the incorporation of a finite velocity of light necessitated a revision of the notion of the simultaneity and of the measurement of time and length intervals. The incorporation of Planck's constant necessitated a revision of the problem of simultaneous measurements of canonically conjugated quantities. It is only natural to expect that the incorporation of a fundamental length should necessitate again a revision of the process of measurement. The theory with a form factor introduces a relativistically invariant action at a distance and, therefore, the measurements on σ_1 and σ_2 may interfere with each other at least for some of the measured quantities called non-localizable.

Since actual measurements take place on the surfaces σ_1 and σ_2 it is natural to express the integral equations (2.7) and (2.11) as initial value problems referring to these surfaces. Thus, (2.7) may be written in the form

$$(4.1)\qquad \Phi^{\varepsilon}(x) = \Phi^{\varepsilon}_{\sigma_2}(x) - \int_{\Omega}\int\int G^{\sigma_2}_{\varepsilon\beta}(x,u)\,\frac{\partial \mathcal{L}'}{\partial \Phi^{\beta}(u)} = \Phi^{\varepsilon}_{\sigma_1}(x) + \int_{\Omega}\int\int G^{\mathrm{ret}}_{\varepsilon\beta}(x-u)\,\frac{\partial \mathcal{L}'}{\partial \Phi^{\beta}(u)}$$

where $\Phi^{\lambda}_{\sigma_1}$ is a solution of the interaction-free equation that coincides (together with the normal derivative) with Φ^{λ} on σ_1. Φ^{β} is again the complex conjugate of Φ^{λ} (in the case of fermions it is the complex conjugate multiplied by γ^4). G^{ret} is defined by

$$G^{\mathrm{ret}}(x) = \begin{cases} -\Delta(x) & \text{for} \quad x_0 \geqslant 0\,, \\ 0 & \text{»} \quad x_0 < 0\,, \end{cases} \tag{4.2}$$

(or a similar expression with $S(x)$ instead of $\Delta(x)$ in the case of fermions). From (2.4) it is seen that $G^{\mathrm{ret}}(x-u)$ is identical with $G^{\sigma_1}(x, u)$ for points x and u in Ω. Analogous integral equations hold for the « initial » conditions referring to the surface σ_2

$$\Phi^{\lambda}(x) - \Phi^{\lambda}_{\sigma_2}(x) + \iiint_{\Omega} G^{\sigma_2}_{(\beta)}(x, u)\, \frac{\partial \mathcal{L}'}{\partial \Phi^{(\beta)}(u)}\, du = \Phi^{\alpha}_{\sigma_2}(x) + \iiint_{\Omega} G^{\mathrm{adv}}_{(\beta)}(x-u)\, \frac{\partial \mathcal{L}'}{\partial \Phi^{\beta}(u)}\, du\,, \tag{4.3}$$

where

$$G^{\mathrm{adv}}(x) = \begin{cases} 0 & \text{for} \quad x_0 > 0\,, \\ \Delta(x) & \text{»} \quad x_0 \leqslant 0\,, \end{cases} \tag{4.4}$$

(or a similar equation with $S(x-u)$). $\Phi^{\lambda}_{\sigma_2}(x)$ is the solution of the interaction free equation coinciding with $\Phi^{\alpha}(x)$ on σ_2. It is easily seen that G^{adv} is identical with G^{σ_2} in the domain Ω.

Direct quantization. Since the free waves $\Phi^{\lambda}_{\sigma_1}$ (or $\Phi^{\lambda}_{\sigma_2}$) determine uniquely the solution of the equation (4.1) or (4.3), we quantize directly the integral field equations replacing the free waves by operators satisfying the usual commutation relations for free waves. In the case of bosons the commutation relations are

$$\left[\Phi^{\alpha}_{\sigma_1}(x),\, \Phi^{\beta}_{\sigma_1}(x')\right] = i\Delta(x-x')\,, \tag{4.5}$$

if $\Phi^{\beta}_{\sigma_1}$ is the complex conjugate of $\Phi^{\lambda}_{\sigma_1}$, otherwise the commutator is zero. In the case of fermions the commutators are replaced by anticommutators and the Δ-function by the S-function [6].

The commutation relations between the perturbed field components follow directly from the integral field equations (4.1) or (4.3) whichs hould be pro-

[6] J. Schwinger: *Phys. Rev.*, **74**, 1439 (1948).

perly symmetrized with respect to the tensor components and antisymmetrized with respect to the spinor components of the fields. The integral equations may be solved by means of iteration. As the zero-order approximation should be taken the operators $\Phi^{\lambda}_{\sigma_1}$ (or $\Phi^{\lambda}_{\sigma_2}$) satisfying the commutation relations (4.5).

Now, question arises as to existence of the solutions of the quantized integral field equations. In the Appendix the proof is given of the existence of the corresponding classical field equations under the assumption that the initial free field components are limited. Here the Φ^{λ}_{σ} are operators and the condition (A.2) loses its sense. Moreover, the commutation relations are singular since the Jordan and Pauli function is singular on the light cone. The question is whether the singular character of the commutation relations (and therefore also of the free field operators) may, or may not spoil the convergence?

We are not able yet to give a rigorous proof but possess strong arguments in favour of the second alternative. First of all we notice the fact that the singular character of the commutation relations does not cause any trouble in the limit of a vanishing interaction ($g \to 0$) where all the results (i.e. the expectation values of observables) are meaningful. Since the kernels of our integral equations are limited, and the domain of integration is finite, we may invoke the arguments of continuity: by taking g small the elements of the matrices describing (in a representation) the perturbed observables would differ by small amounts from the unperturbed ones. The classical condition (A.2) should be replaced by a condition restricting the state vector: we accept only such states that correspond to limited expectation values of observables constructed with the aid of the interaction-free fields $\Phi^{\lambda}_{\sigma_1}$ (or $\Phi^{\lambda}_{\sigma_2}$). Then we may repeat the procedure of iteration and discuss the convergence of the iterated expectation values of observables instead of the iterated field components themselves. (*)

Thus, we have strong arguments in favour of the opinion that the divergences encountered in the traditional quantum field theory should be regarded as classical of origin (i.e. have nothing to do with the « second quantization »). There is no need to change the general scheme of operators and vectors in the Hilbert space nor to make new assumptions for quantization.

Although the general scheme of quantum mechanics remains unchanged, the non-local field model imposes considerable restrictions upon those aspects of the traditional quantum theory which were connected with the localizability of the systems. The most conspicuous difference between the « local » and the « non-local » quantum theory is the absence of a Schrödinger-Tomonaga equation. This may be seen most directly from the fact that the energy

(*) After the completion of this article, sufficient conditions have been formulated for the existence of a solution for the case of a quantized field coupled to an external field. (Compare Appendix II).

of the system is not merely a functional on a surface but depends upon the whole domain in which the system is isolated from interference with the measuring apparatus. The same may be seen also from the fact that the property of additiveness is lacking for a « non-local » action functional

$$W_{\sigma_2,\sigma_1} \neq W_{\sigma_2,\sigma} + W_{\sigma,\sigma_1}, \tag{4.6}$$

which reflects the circumstance that the physical situation is quite different in the two cases when a measurement on an intermediate surface takes place or not. Thus, it is not possible to find a differential characterization of the temporal development of the system. Instead, we possess an integral characterization by means of $W_{\sigma_2\sigma_1}$ and may obtain the desired information concerning the probability amplitudes (connecting the measurements on σ_1 and σ_2) directly from the field equations.

Let us introduce into (4.3) such « initial » wave components $\Phi^\alpha_{\sigma_2}$ which coincide with the solution of (4.1) on the surface σ_2. Then Φ^α are the same solutions of both equations (4.1) and (4.3). By substracting (4.3) from (4.1) we get

$$\Phi^\alpha_{\sigma_2}(x) - \Phi^\alpha_{\sigma_1}(x) = -\iiint_\Omega \mathrm{d}x' \,\mathrm{d}x'' \,\mathrm{d}x''' \Delta(x-u) \frac{\partial \mathcal{L}'}{\partial \Phi^\beta(u)}. \tag{4.7}$$

Since $\Phi^\alpha_{\sigma_1}$ and $\Phi^\alpha_{\sigma_2}$ satisfy the same commutation relations, there exists a unitary matrix U_{21} transforming $\Phi^\alpha_{\sigma_1}$ into $\Phi^\alpha_{\sigma_2}$

$$\Phi^\alpha_{\sigma_2} = U_{21}^{-1} \Phi^\alpha_{\sigma_1} U_{21}. \tag{4.8}$$

The elements of the matrix U_{21} are just the probability amplitudes connected with the development of the system from the initial surface σ_1 to the final surface σ_2. The matrix U_{21} may be computed by means of iteration from (4.1) or (4.3), and (4.7), (4.8). The existence of the matrix U_{21} will be guaranteed by the existence of the solution of the integral field equations.

5. – Some remarks on the singular cases.

We have shown in the foregoing sections that, if the fields are classical, it is possible to construct a regular field theory with solutions which may be computed by means of iteration. Moreover, we have good reasons to believe that the quantization does not spoil the convergence of the field formalism. The proof of the convergence was given under two restrictions: (*i*) a finite

domain limited by two hyper-surfaces where measurements take place, (*ii*) regular kernels of the integral equations. This has been achieved by introducing a suitable form-factor into the interaction part of the action functional. Now, question arises as to whether our restrictions may be removed or replaced by less stringent ones.

The domain Ω may be formally replaced by the whole space time manifold by removing the surfaces σ_1 and σ_2 to the infinite past and future (*). Then, the form-factor (3.16) simplifies to (3.15′) and the computations simplify considerably since we may represent the field components by means of Fourier integrals (7). In the limit of the whole space-time the equations (4.1) and (4.3) become

$$\Phi^{\varkappa}(x) = \Phi^{\varkappa}_{\text{in}}(x) + \iiint G^{\text{ret}}_{(\beta)}(x-u)\,\frac{\partial \mathcal{L}'}{\partial \Phi^{(\beta)}(u)}\,, \tag{5.1}$$

$$\Phi^{\varkappa}(x) = \Phi^{\varkappa}_{\text{out}}(x) + \iiint G^{\text{adv}}_{(\beta)}(x-u)\,\frac{\partial \mathcal{L}'}{\partial \Phi^{\beta}(u)}\,, \tag{5.2}$$

where the integrations extend over the whole space-time manifold while $\Phi^{\varkappa}_{\text{in}}$ and $\Phi^{\varkappa}_{\text{out}}$ mean the free ingoing or outgoing waves. The unitary matrix U_{12} becomes identical with the S-matrix of Heisenberg (8).

It may be easily seen that the non-local S-matrix elements differ from the local ones by the same substitutions which convert the integral field equations of the local field theory into the non-local ones. This offers a possibility of an automatic transcription of the traditional S-matrix into the non-local form. The application of the Fourier analysis to the modified S-matrix amounts to supplying the Fourier integrals by form-factors $F(l_1, l_2, l_3)$ being just the Fourier transforms of the function $F(x', x'', x''')$. In this way it was shown directly (**), (1), (2), (3), that the separate terms of the series expansion of the S-matrix obtained by means of an iteration procedure converge if the form-factor is suitably chosen. However, the proof of the uniqueness of the solution and of the convergence of the whole series was lacking.

Some doubts may be raised against the formal extension of the domain

(*) While the condition (A.1) should be replaced by (A.17).

(7) J. Schwinger: *Phys. Rev.*, **75**, 651 (1949).

(8) C. Yang and D. Felman: *Phys. Rev.*, **79**, 972 (1950).

(**) It may be interesting to notice that the authors version of the reciprocal field theory (1) led to the same prescription for the S-matrix elements. If Fourier analysed they differ from the traditional ones by a set of convergence factors of the type $(\sin^2 a)/a^2$ with $a^2 = (\lambda^4/4)[(\boldsymbol{l}_1 \cdot \boldsymbol{l}_3)^2 - l_1^2 \cdot l_3^2]$ where $\boldsymbol{l}_1$ and $\boldsymbol{l}_3$ are momenta of the electrons. Thus, in spite of quite a different starting point, the reciprocal formalism is equivalent to the formalism with a special form of the form-factor.

of integration to infinity. In this case the initial value problem becomes improper, and question arises as to the meaning of the intuitive phrase: «the free wave coinciding with the solution $\Phi^{\wedge}$ for the infinitely remote past (or future)». The initial value problem at a surface σ tending to infinity is a delicate matter, and, under circumstances, the homogeneous integral equations instead of the inhomogeneous may possess non-vanishing solutions. This possibility is plausible since the statements that $\Delta^{\mathrm{ret}}(x-u)$ vanishes for $x_0 \to -\infty$ and $\Delta^{\mathrm{adv}}(x-u)$ vanishes for $x_0 \to \infty$ are rather intuitive in character since the integration over the variable u is extended over the whole space time manifold. The solutions of the homogeneous integral equations cannot be derived by means of an iteration procedure starting with a free wave as the zero-order approximation. This complicates considerably the problem of a direct quantization of the field equations. The problem of a field theory in an infinite domain is not yet solved and needs further investigations.

Let us discuss briefly the other limit transition $\lambda \to 0$, that is, the transition to the local field theory. In this limit the kernels of the integral field equations become singular. This excludes the existence of solutions in form of a power series in the coupling constant but does not exclude the possibility of existence of singular solutions. It seems that the non-local theory may serve just as a sound basis for investigations of the consistency of the local field theory. If the solutions of the local field equations exist, they should be, in this way or other, continuations to the value $\lambda = 0$ of the solutions with a form factor. The solutions of the non-local field equations for small values of λ may be used as approximations of the solutions of the local field equations (if the latter exist) and in this way the non-local formalism may provide a method of solving approximatively the problems of the local field theory.

The author is much obliged to Professor C. Møller for the opportunity of studying his paper before publication.

Appendix I

A proof of the existence of the solution of the set of the integral equations (2.12).

The kernels K^{σ} (being solutions of the inhomogeneous equations (2.13) with a continuous and limited F in Ω, and satisfying regular initial conditions on σ) are limited

$$|K^{\sigma}| < M . \tag{A.1}$$

Let us assume that the free waves Φ_σ^λ coinciding with the solutions Φ^λ on σ are also limited

$$|\Phi_\sigma^\lambda| \leqslant f \qquad \text{in } \Omega \text{ for any } \alpha . \tag{A.2}$$

We try to solve the equations (2.12) by means of iteration

$$\psi_{n+1}(x) = \psi_\sigma(x) + g \iiint_{\Omega} \mathrm{d}y\, \mathrm{d}z\, K_\psi^\sigma{}^*(x, y, z)\varphi_n(y)\psi_n(z) \tag{A.3}$$

with

$$\psi_0(x) = \psi_\sigma(x) . \tag{A.4}$$

Similar iteration formulae hold for the remaining two integral equations (2.12). The first order iterated function is

$$|\psi_1(x)| \leqslant |\psi_\sigma(x)| + gf^2 \iiint_{\Omega} \mathrm{d}y\, \mathrm{d}z\, |K_\psi^\sigma{}^*(x, y, z)| . \tag{A.5}$$

Let us introduce a number $h > f$ but otherwise arbitrary,

$$|\psi_1(x)| \leqslant f + gf^2 M\Omega^2 < f + gh^2 M\Omega^2 , \tag{A.6}$$

where Ω^2 denotes the square of the volume of the domain Ω. The same inequality holds for the other two equations whence the iterated first order functions are all limited

$$|\psi_1| < h , \qquad |\varphi_1| < h , \tag{A.7}$$

if

$$g \leqslant \frac{h - f}{h^2 M\Omega^2} < \frac{1}{4fM\Omega^2} . \tag{A.8}$$

Taking the second order iteration formulae, we find again

$$|\psi_2| \leqslant f + gh^2 M\Omega^2 , \tag{A.6'}$$

so that the same restriction on the coupling constant holds. The same procedure may be repeated ad infinitum. In this way it is proved that all the iterated functions are limited by the same $h > f$

$$|\psi_n| \leqslant h , \qquad |\varphi_n| \leqslant h , \tag{A.9}$$

if g is sufficiently small.

In order to show the convergence of the sequence ψ_n, φ_n we substract the

n-th iterated formula from the $(n+1)$-th

$$|\psi_{n+1}-\psi_n| \leqslant gM \iint_{\Omega} \mathrm{d}y\,\mathrm{d}z\,|\varphi_n\psi_n-\varphi_{n-1}\psi_{n-1}| \leqslant$$

$$\leqslant gM \iint_{\Omega} \mathrm{d}y\,\mathrm{d}z\,|\varphi_n(\psi_n-\psi_{n-1})+(\varphi_n-\varphi_{n-1})\psi_{n-1}|\,,$$

or

$$|\psi_{n+1}-\psi_n| \leqslant ghM \iint_{\Omega} \mathrm{d}y\,\mathrm{d}z\,(|\psi_n-\psi_{n-1}|+|\varphi_n-\varphi_{n-1}|)\,. \tag{A.10}$$

In the same way

$$|\varphi_{n+1}-\varphi_n| \leqslant 2ghM \iint_{\Omega} \mathrm{d}y\,\mathrm{d}z\,|\psi_n-\psi_{n-1}|\,, \tag{A.10'}$$

while

$$|\psi_1-\psi_0| \leqslant a\,, \qquad |\varphi_1-\varphi_0| \leqslant a\,, \tag{A.11}$$

with

$$a = gh^2M\Omega^2\,, \tag{A.11'}$$

whence

$$|\psi_{n+1}-\psi_n| \leqslant (2ghM\Omega^2)^n a\,, \qquad |\varphi_{n+1}-\varphi_n| \leqslant (2ghM\Omega^2)^n a\,. \tag{A.12}$$

This result shows that the sequences ψ_n and φ_n are uniformly convergent if

$$g \leqslant \frac{1}{2h\overline{M\Omega^2}}, \tag{A.13}$$

or

$$g \leqslant \frac{1}{4f\overline{M\Omega^2}}, \tag{A.13'}$$

(since $2f = h$ is the optimal case). The limes of the sequences represents a solution.

It may be easily shown that there exists no other limited solution satisfying the same initial conditions. Let us assume that there exists a solution $\psi \neq \lim_{n\to\infty} \psi_n$ with $|\psi| < h'$. Denoting $\max(h, h')$ by H we find easily

$$|\psi-\psi_n| \leqslant 2gMH \iint \mathrm{d}y\,\mathrm{d}z\,|\psi-\psi_{n-1}|\,, \tag{A.14}$$

and

$$|\psi - \psi_0| \leqslant gH^2 M\Omega^2 , \tag{A.15}$$

whence

$$\lim_{n\to\infty} |\psi - \psi_n| = 0 , \tag{A.16}$$

contrary to the assumption.

The appearance of $M\Omega^2$ in (A.13) shows that the condition (A.1) may be replaced by

$$\iint_{\Omega} |K(x, y, z)| \, dy \, dz \leqslant M . \tag{A.17}$$

Appendix II (*)

The case of a quantized field ψ in a given external field φ.

The equations (2.12) reduce in this case to

$$\psi(x) = \psi_\sigma(x) + g \int_{\Omega} dy \, N(x, y)\psi(y) , \tag{A.18}$$

and a similar equation for the hermitian conjugate ψ^*. The kernel of the integral equation (A.18) is

$$\left| \begin{array}{l} N(x, y) = \iint_{\Omega} G^{\sigma}_{(m)}(x, x')F(x', x'', y;\ \Omega)\varphi(x'') \, dx' \, dx'' , \\ N^*(x, y) = \iint_{\Omega} G^{\sigma}_{(m)}(x, x''')F(y, x'', x''';\ \Omega)\varphi(x'') \, dx'' \, dx''' . \end{array} \right. \tag{A.19}$$

The equation (A.18) and its complex conjugate is a symbolic equation between operators and may be replaced by a set of usual equations (in terms of c-numbers) when introducing arbitrary state vectors $|\alpha'\rangle$, $|\beta'\rangle$, etc.

$$\langle\beta'|\psi(x)|\alpha'\rangle = \langle\beta'|\psi_\sigma(x)|\alpha'\rangle + g \int_{\Omega} dy \, N(x, y)\langle\beta'|\psi(x)|\alpha'\rangle , \tag{A.18'}$$

and

$$\langle\alpha'|\psi^*(x)|\beta'\rangle = \langle\alpha'|\psi^*_\sigma(x)|\beta'\rangle + g \int_{\Omega} dy \, N^*(x, y)\langle\alpha'|\psi^*(y)|\beta'\rangle . \tag{A.18''}$$

(*) Added in proof.

These are linear integral equations which may be solved in the case of a limited kernel $N(x, y)$. The zero order functions $\langle\beta'|\psi_\sigma(x)|\alpha'\rangle$ need not to be limited but the integral

$$I(x) = \int_{\Omega} \mathrm{d}y\, |N(x, y)\langle\beta'|\psi_\sigma(y)|\alpha'\rangle| \tag{A.20}$$

must exist. The solution is

$$\langle\beta'|\psi(x)|\alpha'\rangle = \langle\beta'|\psi_\sigma(x)|\alpha'\rangle + g\int_{\Omega} \mathrm{d}y\, R(x, y; g)\langle\beta'|\psi_\sigma(y)|\alpha'\rangle\,, \tag{A.21}$$

where $R(x, y; g)$ is the resolving kernel satisfying the equation

$$R(x, y; g) = N(x, y) + g\int_{\Omega} \mathrm{d}z N(x, z)R(z, y; g). \tag{A.22}$$

For small values of g the resolving kernel admits a power series expansion

$$R(x, y; g) = N(x, y) + \sum_{n=1}^{\infty} g^n N_n(x, y)\,, \tag{A.23}$$

where

$$N_n(x, y) = \int_{\Omega} \mathrm{d}z_1 \ldots \int_{\Omega} \mathrm{d}z_n N(x, z_n)N(z_n, z_{n-1}) \ldots N(z_1, y)\,. \tag{A.23'}$$

The series (A.23) converges vor $g < 1/M\Omega$ where $M \geqslant |N(x, y)|$. In order to have a limited kernel, the external field φ does not need to be limited itself since the form factor F may secure the convergence even in the case of a singular function φ.

Quantum mechanics is chiefly interested in the expectation values of observable densities. We form

$$\begin{aligned}\langle\alpha'|\psi^*(x)\psi(x)|\alpha'\rangle &= \sum_{\beta'}\langle\alpha'|\psi^*(x)|\beta'\rangle\langle\beta'|\psi(x)|\alpha' \quad = \\ &= \langle\alpha'|\psi_\sigma^*(x)\psi_\sigma(x)|\alpha'\rangle + g\int_{\Omega} \mathrm{d}y\, R(x, y; g)\langle\alpha'|\psi_\sigma^*(x)\psi_\sigma(y)|\alpha'\rangle + \\ &+ g\int_{\Omega} \mathrm{d}y\, R^*(x, y; g)\langle\alpha'|\psi_\sigma^*(y)\psi_\sigma(x)|\alpha'\rangle + \\ &+ g^2\iint_{\Omega} \mathrm{d}y\, \mathrm{d}z\, R^*(x, y; g)R(x, z; g)\langle\alpha'|\psi_\sigma^*(y)\psi_\sigma(z)|\alpha'\rangle\,.\end{aligned} \tag{A.24}$$

Denoting by $\varrho(x)$ any observable density (being a bilinear form in $\psi(x)$, $\psi^*(x)$ and their first derivatives) and by $\varrho(x, y)$ the corresponding quantity obtained

from $\varrho(x)$ by replacing $\psi^*(x)$ by $\psi^*(y)$ (and $\partial_\mu\psi^*(x)$ by $\partial_u\psi^*(y)$) we have also

$$\text{(A.24')} \qquad \langle\alpha'|\varrho(x)|\alpha'\rangle = \langle\alpha'|\varrho_\sigma(x)|\alpha'\rangle + g\int_\Omega dy\, R(x,y;g)\langle\alpha'|\varrho_\sigma(y,x)|\alpha'\rangle +$$

$$+g\int_\Omega dy\, R^*(x,y;g)\,\langle\alpha'|\varrho_\sigma(x,y)|\alpha'\rangle + g^2\iint_\Omega dy\,dz\, R(x,y;g)R^*(x,z;g)\,\langle\alpha'|\varrho_\sigma(y,z)|\alpha'\rangle,$$

expressing the perturbed density in terms of the corresponding bi-density formed of the unperturbed operators ψ_σ, ψ_σ^* and their first derivatives, and of the resolving kernel. The expression (A.24') is meaningful if the integrals to the right hand side are meaningful. Let us consider, for example, the second integral in (A.24')

$$\text{(A.25)} \qquad J(x) = \int_\Omega dy\, R(x,y;g)\langle\alpha'|\varrho_\sigma(y,x)|\alpha'\rangle\,.$$

Denoting bu $I(y,x)$ the quantity

$$\text{(A.26)} \qquad I(y,x) = \int_\Omega dz N(y,z)\langle\alpha'|\varrho_\sigma(z,x)|\alpha'\rangle\,,$$

we have (with the aid of (A.23))

$$\text{(A.727)} \quad J(x) = I(x,x) + g\int dy\, N(x,y)I(y,x) + g^2\int dy\, N_1(x,y)I(y,x) + \ldots =$$

$$= I(x,x) + g\int_\Omega dy\, R(x,y;g)I(y,x)\,.$$

Since the resolving kernel R is limited, the existence of a limited integrand

$$\text{(A.28)} \qquad |I(x,y)| \leqslant A\,,$$

where $I(x,y)$ is given by (A.26) is a sufficient condition for the consistency of the quantum theory of fields coupled to an external field.

The condition (A.28) is a restriction upon the kernel $N(x,y)$, that is, upon the form factor F, but may restrict also the class of state vectors.

The above considerations may be extended to the case of arbitrary values of the coupling parameter g. To this end we have to use the theory of Fredholm. In general, the resolving kernel is a quotient of two integer functions of the coupling parameter g. The question of the existence of the poles of the kernels is to be investigated.

RIASSUNTO (*)

L'A. costruisce una teoria regolare dei campi in un dominio Ω, finito, dello spazio-tempo, delimitato da due ipersuperficie spaziali. Le equazioni del campo sono equazioni integrali con noccioli regolari. Le soluzioni sono funzioni intere del parametro di accoppiamento. Se ne deducono leggi di conservazione integrale. Onde assicurare la corrispondenza con la tradizionale teoria dei campi, il fattore di forma (introdotto per normalizzare i noccioli) deve dipendere dal dominio Ω. Le equazioni integrali del campo si quantizzano direttamente senza far ricorso al formalismo canonico. L'equazione di Schrödinger-Tomonaga non esiste, tuttavia il formalismo fornisce risultati esenti da ambiguità sulle ampiezze di probabilità colleganti le misure sulle due ipersuperficie. L'A. è del parere che la quantizzazione dei campi non ha alcuna relazione con le difficoltà di far convergere le espressioni trovate. Queste scompaiono nel caso di un dominio finito e di noccioli regolari.

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Spinor relativity.

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Summary. — The pure-geometrical part of the paper makes various contributions to the theory of metric and connection in spinspace. The physical part gives a unified field theory, which aims to include the matter field, obtained by putting a simple (Einsteinian) condition on the curvature of the spin connection used. Of the resulting theory we mention in particular that it generalizes the author's Conformal Relativity, that it admits the conformal group as physical group, and that the theory of matter so obtained is of Dirac form.

Introduction.

This paper falls into two main parts, a pure geometrical part (sections **1** - **3**) making contributions to spin-space theory, and another part (section **4**) postulating a unified field theory which generalizes the author's Conformal Relativity [1] with the aim of geometrizing matter.

We are given a n- (or $(n-1)$-) dimensional manifold ($n = 2\nu$) with real affine (or real projective) local spaces E_n (or P_{n-1}). It bears also local spin spaces S_{2^ν}: complex projective space $P^*_{2^\nu-1}$ provided with a set of n involutions $\eta^{\varrho a}{}_b(x^\mu)$ (1), or η^ϱ in matrix notation, satisfying

$$\eta^{(\mu}\eta^{\varrho)} = \delta^{\mu\varrho}A \qquad \text{[matrix multiplication]} \tag{1}$$

(1) Small greek letters λ, μ, ν, ..., $\omega = 1, ..., n$; small latin letters a, b, c, ..., $g = 1, ..., 2^\nu$. (μ) denotes a (holonomic or non-holonomic) system of local coordinate systems in the local E_n's (or P_{n-1}'s); x^μ are the manifold coordinates; (a) denotes a system of local coordinate systems in the local S_{2^ν}'s. Capital greek letters A, M, N, ..., Ω run through the 2^n combinations of the integers $(1, ..., n)$ p at a time $[p = 0, 1, ..., n]$: Φ_0, $\Phi_1 = \varphi$, $\Phi_2 = [\varphi_1\varphi_2]$, $\Phi_3 = [\varphi_1\varphi_2\varphi_3]$, ..., $\Phi_n = [\varphi_1 ... \varphi_n]$. Further alphabets will be introduced as needed.

A ($= A^{a}_{b}$) the unit tensor of S_{2^ν}. The Clifford algebra of the η^ϱ brings with it an array of fundamental automorphisms [2] which will play an important role below. For convenience we will « normalize » $P^{*}_{2^\nu-1}$ by considering it an affine space $E^{*}_{2^\nu}$; the effect of the transformation $\chi^{a} \to \lambda\chi^{a}$ can then be obtained by an affine transformation $A^{a'}_{a} = \delta^{a'}_{a}\lambda$. The systems (a) and (a') are then to be considered not essentially different from the projective viewpoint. Section 1 deals with properties independent of the connection of local spaces, hence we can drop the adjective « local » throughout it without ambiguity. In many places we specialize to $\nu = 3$, local P_5's and S_8's, and the signature $(-++\,+--+)1$ of $S_{\mu\varrho}$ in view of the application to Spinor Relativity, but the corresponding treatment for general ν will be evident.

Matrix conventions for suppressing spin-indices will be used. The marks $^{-}$, $^{\sim}$, and † denote the complex conjugate $\overline{\eta}^{\varrho\dot{a}}_{\dot{b}}$ $(\overline{\eta^{\varrho a}_{b}})$, transpose $\widetilde{\eta}^{\varrho}_{b^{a}} \equiv \eta^{\varrho a}_{b}$, and hermitian conjugate ${}^{\dagger}\eta^{\varrho}_{\dot{b}}{}^{\dot{a}}$ $: (\widetilde{\eta}^{\varrho}_{b^{a}}) = (\eta^{\varrho a}_{b})$ resp., correspondingly for spin tensors of other type. Dotted indices $\dot{a}$ transform with the complex conjugate transformation $A^{\dot{a}'}_{\dot{a}} \equiv (A^{a'}_{a})$.

1. – The Metric.

For any vector χ^{a} of S_8, the expression

$$g_{\dot{a}b}\overline{\chi}^{\dot{a}}\chi^{b} = (\chi)^{2},$$

where $g_{\dot{a}b}$ is the hermitian anti-hermitizing tensor (2)

$$g\eta^{\varrho}g^{-1} = -{}^{\dagger}\eta^{\varrho} \tag{1.1}$$

is real, and will be defined as the square of the length of χ (3). Then if we regard Π $(\Pi^{\dot{a}}_{b})$, the complex conjugating tensor (2)

$$\Pi\eta^{\varrho}\Pi^{-1} = \overline{\eta}^{\varrho}, \tag{1.2}$$

as defining a *change of system* $(a) \to (\dot{a})$ given by $A^{\dot{a}}_{b} = \Pi^{\dot{a}}_{b}$, then $g_{\dot{a}b}$ appears

(2) SCHOUTEN, loc. cit., pp. 219-221. We emphasize that since we want to retain complete generality in the transformation group, we shall *not* subject these automorphisms to the various normalizations introduced by SCHOUTEN which change them from spin tensors into spin tensor densities.

(3) The choice of the hermitizing automorphism as metric leads to a skew metric for $\nu = 3$.

as the mixed components between (a) and $(\dot{a})$ of the metric; and we can obtain all other mixed and unmixed components $g_{a\dot{b}}$, $g_{\dot{a}\dot{b}}$, $g_{\dot{a}\dot{b}}$ by transforming it tensorially with Π. E.g., $g_{ab} = A_a^{\dot{a}} g_{\dot{a}b} \equiv \bar{\Pi}_a^{\dot{a}} g_{\dot{a}\dot{b}}$, etc., g_{ab} is in fact the negative transposing tensor (2):

$$g\eta^{\varrho}\bar{g}^{-1} = -\bar{\eta}^{\varrho} \tag{1.3}$$

and is symmetric for $r = 3$ (4). Then the equation defining $g_{\dot{a}\dot{b}}$'s inverse

$$g\bar{g}^{-1} = \bar{g}^{-1}g = A\,, \tag{1.4}$$

is true by definition for any set of mixed or unmixed components.

A second way of defining further components of the metric would have been by complex conjugation: from $G_{\dot{a}\dot{b}} = g_{\dot{a}\dot{b}}$ and $G_{ab} = g_{ab}$ we can define

$$G_{a\dot{b}} = (G_{\dot{a}b}) \qquad G_{a\dot{b}}\,, \qquad G_{\dot{a}\dot{b}} \qquad (G_{ab}) = G_{\dot{a}\dot{b}}\,.$$

The consistency theorem of this calculus is then

$$G = g\,, \qquad \text{[any set of components]} \tag{1.5}$$

which is easily verified using the facts $g_{\dot{a}\dot{b}}$ is hermitian, and for $r = 3$ g_{ab} is symmetric and $\Pi^{\dot{a}}_{\ b}$ for the signature of our $S_{\mu\varrho}$, is *positive unitary* $(\bar{\Pi} = \bar{\Pi}^{-1})$ (5). Thus in particular we have a symmetric metric (where in the mixed components, this manifests itself as *hermitian* symmetry). I.e., complex conjugation can be treated simply as a change of system as far as the metric is concerned. (1.2) then shows that this is also true for the further class of tensors η^{ϱ}. Accordingly we can omit the bar of complex conjugation over the kernel of the η^{ϱ} and g, and (1.1) is comprised in (1.3). Equations (1.3) and (1.4) thus summarize all the relations among the η^{ϱ} and the three basic automorphisms $g_{\dot{a}\dot{b}}$, g_{ab}, $\Pi^{\dot{a}}_{\ b}$ and their inverses.

This suggests immediately the (invariant) notion of a *real spintensor* as a natural generalization of a real spinscalar $\omega = \bar{\omega}$. We define: $T^{\dot{a}\dots}_{\ \dot{b}\dots}$ is a *real tensor* if under complex conjugation it transforms tensorially by means of $A^{\dot{a}}_{b} \equiv \Pi^{\dot{a}}_{\ b}$

$$(T^{\dot{a}\dots}_{\ \dot{b}\dots}) \equiv \bar{T}^{a\dots}_{\ \ b\dots} = A^{ab\dots}_{\dot{a}\dot{b}\dots} T^{\dot{a}\dots}_{\ \dot{b}\dots}\,. \tag{1.6}$$

(4) SCHOUTEN, loc. cit., p. 340.

(5) SCHOUTEN, loc. cit., p. 342. « Positive invertible », following SCHOUTEN.

For, viewing Π as a transformation of system, (1.6) states that the tensors $\bar{T}$ and T are identical. The reason for requiring the $A^{\dot{a}}_{b}$ in (1.6) to be specifically $\Pi^{\dot{a}}_{\dot{b}}$, not just any functions, is that the transformation $(a) \to (\dot{a})$ must be given by Π by (1.2). E.g., g, Π, and η^{ϱ} are real tensors (N.B. $\bar{\bar{\Pi}} = + \bar{\Pi}^{1}(\Pi\bar{\Pi})^{-1} = \bar{A}^{-1}\Pi\bar{A}$, and is thus real). The connection with the term « real » in the sense of real numbers, is this: every real tensor has real components in some system. Proof: for Π is positive unitary, thus $\Pi^{\dot{a}}_{\dot{b}} = \delta^{\dot{a}}_{\dot{b}}$ in some system (a) [6]. One can now define real more general geometric objects in the obvious way. For example, we shall have to do presently with a real spinvector connection.

With our metric we can now raise and lower vector indices. Writing $(\chi)^2$ first as $\chi_a\chi^a$ and then as $\bar{\chi}_{\dot{a}}\bar{\chi}^{\dot{a}}$, we must define

$$\chi_a \equiv g_{\dot{b}a}\bar{\chi}^{\dot{b}}, \qquad \bar{\chi}_{\dot{a}} \equiv g_{\dot{a}b}\chi^{b}. \tag{1.7}$$

These are equivalent in virtue of the reality of g. Left multiplying by $\bar{g}^{-1}$ we get the rules for raising vector indices. In addition we use Π as defining the change of system $(a) \to (\dot{a})$: e.g., $\chi^{\dot{a}} = A^{\dot{a}}_{b}\chi^{b} = \Pi^{\dot{a}}_{\ b}\chi^{b}$. The the calculus of raising and lowering and « switching » (dotted and undotted) indices defined by (1.7) and the last sentence is consistent under complex conjugation. This follows immediately from the fact that g and Π are real tensors. In matrix notation (1.7) becomes

$$\chi = g\bar{\chi}, \qquad \bar{\chi} = g\chi. \tag{1.7$'$}$$

Now we will single out for special attention one particular vector ψ and retain this kernel for it throughout. With the aid of the $\eta^{\Phi_0} = A$, $\eta^{\Phi_p} = \eta^{[\varphi_1}\eta^{\varphi_2} \ldots \eta^{\varphi_p]}$, $[p = 1, \ldots, n]$ we can form the 2^n contravariant spinvector$-p$ vectors ψ^{Φ} $(= \psi^{\Phi a})$ and the 2^n covariant spinvector$-p$ vectors Ψ^{Φ} $(\equiv \Psi^{\Phi}{}_{a})$ given by

$$\psi^{\Phi} \equiv \eta^{\Phi}\psi, \qquad \Psi^{\Phi} \equiv \psi\eta^{\Phi}, \tag{1.8}$$

where $\psi\eta^{\Phi}$ means $\psi_b\eta^{\Phi b}{}_{a}$ of course [1]. Define the covariant components of ψ^{Φ}

[6] Schouten, loc. cit., p. 342. From little theorem flow the obvious corollaries: 1. Conversely, any tensor with real components in a system (a) for which $\Pi^{\dot{a}}_{\dot{b}} = \delta^{\dot{a}}_{\dot{b}}$ is a real tensor. 2. One such (a) can be found in which all real tensors assume real components. 3. All other such systems are got from such an (a) by transformations leaving Π invariant.

and $\overline{\psi}^{\Phi}$ by

$$\psi^{\Phi_p} \equiv (-1)^{S(p)} g \overline{\psi}^{\Phi_p}, \qquad \overline{\psi}^{\Phi_p} \equiv (-1)^{S(p)} g \psi^{\Phi_p} \qquad [S(p) \equiv \tfrac{1}{2} p(p+1)] \tag{1.9}$$

which reduces to (1.7)′ for $p = 0$. The consistency theorem for this further calculus runs

$$\psi^{\Phi} = \Psi^{\Phi}, \qquad \overline{\psi}^{\Phi} = \overline{\Psi}^{\Phi}. \tag{1.10}$$

I.e., the operation of raising and lowering spinvector – p vector indices and that of transforming (contravariant or covariant) spinvectors by the special collineations η^{Φ} commute. In proof, first we note

$$g \eta^{\Phi_p} \bar{g}^1 = (-1)^{S(p)} \widetilde{\eta}^{\Phi_p}, \tag{1.11}$$

which follows from (1.3). Then $\Psi^{\Phi_p} \equiv \psi \eta^{\Phi_p} = \widetilde{\eta}^{\Phi_p} g \overline{\psi} = (-1)^{S(p)} g \eta^{\Phi_p} \overline{\psi} = = (-1)^{S(p)} g \psi^{\Phi_p} \equiv \psi^{\Phi_p}$, which was to be shown. Similarly for the second part. From (1.9) we get rules for raising indices by left multiplying by $(-1)^{S(p)} g^1$. Then this calculus of raising, lowering, and switching the spin indices of spinvector – p vectors is consistent also under complex conjugation, as follows immediately from the corresponding consistency for simple spinvectors ($p = 0$). The geometrical interpretation of these two consistency properties will not be treated here.

The tensors M^{Φ} (scalar as to spin transformations) defined

$$M^{\Phi} \equiv \psi \psi^{\Phi} = \psi^{\Phi} \psi, \tag{1.12}$$

(where by convention, in a product of ψ^{Φ}'s, the first factor will *always* have the covariant index: e.g., $\psi \psi^{\Phi} \equiv \psi_a \psi^{\Phi a}$) have simple reality properties. In fact $\overline{M}^{\Phi_p} = \overline{\psi}\, \overline{\psi}^{\Phi_p} = (-1)^{S(0)+S(p)} \psi^{\Phi_p} \widetilde{\bar{g}}^1\, g \psi = (-1)^{S(p)} \psi^{\Phi_p} \psi \equiv (-1)^{S(p)} M^{\Phi_p}$, using the symmetry of the metric. Thus

$$\overline{M}^{\Phi_p} = (-1)^{S(p)} M^{\Phi_p}. \tag{1.13}$$

Hence M^{Φ_p} is real for $p = 0, 3, 4$, pure imaginary for $p = 1, 2, 5, 6$.

2. – The Connection.

The covariant derivative of spinvectors fixing the η^{ϱ}'s and determined only modulo the unit tensor which has been found by many authors, e.g. [3], can be arrived at elegantly in a new way via the well determined spin-col-

lineation derivative [4], as will be done below. But other authors then got around the indeterminacy of the connection by considering only tensor densities of certain weights [3] whose covariant derivatives involved only the well-determined traceless part (not a connection!) of the spin connection, this procedure being equivalent essentially to restricting the group in S_{2^ν} by certain normalization (e.g., Det $A_b^{a'} = \text{const.}$) (7). We prefer to retain complete generality in the transformation group and at the same time to determine completely a covariant derivative of spin vectors. *This can be done by demanding that the connection fix not only the η^ϱ but all the fundamental automorphisms of its Clifford algebra.*

We now assume that the base manifold is provided with the symmetric (affine or projective) connection $\Gamma_{\varrho\pi}^{\ \ \varphi}$. Then this connection induces a connection $\Gamma_{\psi\Pi}^{\ \ \Phi}$ of the local $2^n - 1$ dimensional projective space S_{2^n} of spin collineations, as explained in [4]. Of this theory we need only recall here the following. We know that

$$\Gamma_{\varrho A}^{\ \ B} = \mathscr{A}_{A\Phi}^{\Pi B}\Gamma_{\varrho\Pi}^{\ \ \Phi}\,, \qquad \text{[canonical]} \tag{2.1}$$

are the (1-vector) components wrt (A) (« with respect to (A) ») of this connection of S_{2^n}. Thereby: A, B, C, ..., G are collineation indices, $A = ({}^{a_1}_{a_2})$, with the convention $T_A = T^{a_1}_{\ \ a_2}$, $T^A = T^{a_2}_{\ \ a_1}$; [canonical] means that the systems (φ) and (a) are *canonically related*, that is, so that the η^φ, wrt (φ) and (a) are certain constant matrices over the manifold (so that $S_{\mu\varrho}$ are also constant); $\Gamma_{\varrho\Pi}^{\ \ \Phi}$ is determined in terms of $\Gamma_{\varrho\pi}^{\ \ \varphi}$ as

$$\Gamma_{\varrho\Pi_p}^{\ \ \Phi_q} = \delta_p^q\, \mathrm{A}^{\varphi_1\ldots\varphi_{p-1}}_{[\pi_1\ldots\pi_{p-1}}\Gamma_{\varrho|\pi_p]}^{\ \ \varphi_p\}}\,;$$

$\mathscr{A}_A^\Phi$ is the special transformation in S_{2^n} from a substratum system (Φ) to a spin collineation system (A) defined by

$$\mathscr{A}_A^{\Phi_p} \equiv \eta^{\Phi_p}{}_A\,, \qquad \mathscr{A}_{\Phi_p}^{A} \equiv (-1)^{\binom{p}{2}}\,{}^1\!/_{2^\nu}\eta_{\Phi_p}{}^A \qquad [p = 0, 1, \ldots, n] \tag{2.2}$$

where Φ in η_Φ is lowered by means of $S_{\mu\varrho}$.

Remark: In a canonical frame $\{(\varphi), (a)\}$, the $\mathscr{A}_A^\Phi$ and $\mathscr{A}_\Phi^A$ are constants, hence the equations $\nabla_\varrho \mathscr{A}_A^\Phi = 0$ defining the new (1-vector) components yield simpy (2.1).

This said, if there is a spin vector connection $\Gamma_\varrho^{\ a_1}_{\ \ a_2}$ fixing η^μ:

$$\nabla_\varrho \eta^\mu \equiv \partial_\varrho \eta^\mu + \Gamma_{\varrho\nu}^{\ \ \mu}\eta^\nu + \Gamma_\varrho \eta^\mu - \eta^\mu \Gamma_\varrho = 0\,, \tag{2.3}$$

(7) Schouten, loc. cit. ante, p. 221.

then it also fixes all the η^{Φ} and $S^{\mu\varrho}$:

$$\nabla_\varrho \mathscr{R}_A^{\Phi} = 0\,, \qquad \Gamma_{\varrho\nu}{}^{\mu} = \left\{ \begin{matrix} \mu \\ \varrho\nu \end{matrix} \right\}_S . \tag{2.4}$$

Then by (2.4) Γ_ϱ must determine a collineation connection $\overset{S}{\Gamma}_{\varrho A}{}^{B}$ corresponding to $\overset{S}{\Gamma}_{\varrho\nu}{}^{\mu} \equiv \left\{ \begin{matrix} \mu \\ \varrho\nu \end{matrix} \right\}_S$ according to

$$\overset{S}{\Gamma}_{\varrho A}{}^{B} = A^{a_1}_{b_1} \Gamma_{\varrho}{}^{b_2}{}_{a_2} - A^{b_2}_{a_2} \Gamma_{\varrho}{}^{a_1}{}_{b_1} . \tag{2.5}$$

$\Gamma_{\varrho}{}^{a_1}{}_{a_2}$ is determined by (2.5) only up to a term of the form $V_\varrho A^{a_1}_{a_2}$. We write $\Gamma_{\varrho}{}^{a_1}{}_{a_2} = \Lambda_{\varrho}{}^{a_1}{}_{a_2} + V_\varrho A^{a_1}_{a_2}$ and normalize Λ_ϱ by demanding $\Lambda_\varrho{}^{a}{}_{a} = 0$. Then (2.5) gvies, contracting a_1, b_1, then a_2, b_2:

$$\Lambda_{\varrho}{}^{a_1}{}_{a_2} = \tfrac{1}{2\nu} \Gamma_{\varrho}{}^{c}{}_{a_2}{}^{a_1}{}_{c} = -\tfrac{1}{2\nu} \Gamma_{\varrho}{}^{a_1}{}_{c}{}^{c}{}_{a_2} . \tag{2.6}$$

Evaluating this for simplicity in a canonical frame using (2.1), one gets after some manipulation ($\nu = 3$, the case of interest)

$$\Gamma_\varrho - V_\varrho A = \Lambda_\varrho = -\tfrac{1}{4} \overset{S}{\Gamma}_{\varrho\pi\lambda} \eta^{[\pi\lambda]} \qquad \text{[canonical]}. \tag{2.7}$$

That is, if such a Γ_ϱ existed, it would be of this form. But now it can be verified directly that this Γ_ϱ (V_ϱ arbitrary) does indeed satisfy (2.3).

It remains only to determine V_ϱ. Choosing a canonical frame, then by the fundamental theorem of the automorphisms of a Clifford algebra (8), g ($= g_{\dot a \dot b}$) and Π in this frame must be constant tensors δ and π resp. up to factors (which are functions of position over the manifold):

$$g = \sigma^2 \delta\,, \quad [\sigma \text{ real}]; \qquad \Pi = \exp[-2i\theta]\pi, \quad [\theta \text{ real}]\,, \tag{2.8}$$

where the reality of σ and θ follow from the (invariant) equations ${}^{\dagger}g = g$, $\overline{\Pi} = \pm \Pi^{\dagger}$, and the permissible choice ${}^{\dagger}\delta = \delta$, $\bar\pi = \bar\pi^{-1}$. First, complex conjugate (2.7):

$$\bar\Gamma_\varrho - \bar V_\varrho A = \Pi(\Gamma_\varrho - V_\varrho A)\bar\Pi^{-1}, \qquad \text{[canonical]} \tag{2.9}$$

(8) SCHOUTEN, loc. cit. ante, p. 182.

which, right multiplied by Π, can be written

$$(2.9)' \qquad \nabla_\varrho \Pi - \pi\{\partial_\varrho(\exp[-2i\theta]) + \exp[-2i\theta]2i \operatorname{Im} V_\varrho\} = 0 \quad \text{[canonical]}.$$

Next, apply ∇_ϱ to g in this frame

$$(2.10) \qquad \nabla_\varrho g = \delta\partial_\varrho\sigma^2 + \tfrac{1}{4}\overset{S}{\Gamma}_{\varrho\pi\lambda}(g\eta^{[\pi\lambda]} + \tilde{\eta}^{[\pi\lambda]}g) - 2g \operatorname{Re} V_\varrho =$$

$$= \delta(\partial_\varrho\sigma^2 - 2\sigma^2 \operatorname{Re} V_\varrho), \quad \text{[canonical]}$$

the bracketed term vanishing by (1.11) for $p = 2$. Then V_ϱ is determined completely by the demand that our connection fix also g and Π:

$$(2.11) \qquad a. \quad \nabla_\varrho \Pi = 0, \qquad b. \quad \nabla_\varrho g = 0\ ;$$

we get $V_\varrho = \partial_\varrho \log(\sigma e^{i\theta})$ in this frame (a). Then its value $V_\varrho{}^{(\prime)}$ [9] in any other system (a') is determined by its transformation rule

$$(2.12) \qquad V_\varrho{}^{(\prime)} = V_\varrho + \partial_\varrho \log \Delta^{2-\nu} = \partial_\varrho \log(\tau\Delta^{2-\nu}) \qquad [\tau \equiv \sigma e^{i\theta},\ \Delta \equiv \operatorname{Det} A_b^{a'}].$$

Carrying out the simple transformation $A_b^{a'} = \delta_b^{a'}\tau$, the components of η^ϱ are unchanged, g and Π become the constant tensors δ and π resp., and $V_\varrho{}^{(\prime)} = 0$ in (a'). (a') is thus also related canonically to (ϱ); when we speak of canonical frame in the future, we will mean, for a given (ϱ), that unique (a') satisfying these additional conditions.

From (2.11) it follows that Γ_ϱ fixes also the other components of the metric (the negative transposing g_{ab} in particular). The only other fundamental automorphism to consider is R $(\equiv R^a{}_b) \propto \eta^{\Phi_n}$ [10] which obeys

$$R\eta^\varrho R^{-1} = -\eta^\varrho\ .$$

If we normalize by putting simply $R = \eta^{\Phi_n}$, then R too is automatically fixed by our connection . Then the dual array of fundamental automorphisms obtained by multiplying those considered above by R (the positive hermitizing, positive transposing, negative complex conjugating [11]) are all fixed by our connection. Summing up, we see that Γ_ϱ fixes the η^ϱ and all their fundamental automorphisms, and is uniquely determined thereby.

By (2.11) a. Γ_ϱ is a *real spinconnection*, i.e., by definition transforms like

[9] The parentheses around the prime are necessitated because V_ϱ bears no spin indices.

[10] Schouten, loc. cit. ante, p. 219. Note that R is thus a pseudoscalar wrt transformations $(\varphi) \to (\varphi')$.

[11] Schouten, loc. cit. ante, pp. 219-221.

a connection on complex conjugation by means of the transformation $A^{\dot{a}}_{b} \equiv \Pi^{\dot{a}}_{b}$. For real spinconnections also, it is true that they have real components in some frame $\{(\varrho), (a)\}$. Real spinconnections take real tensors into real tensors.

3. – Curvature.

The above covariant derivative is now expanded to a derivative with a spin collineation index by the method of p-vector covariant differentiation [4]:

$$\nabla_{\Phi_p}\chi \equiv \nabla^{p}_{[\varphi_1 \ldots \varphi_p]}\chi\,, \qquad \nabla_{\Phi_0}\chi \equiv \chi \tag{3.1}$$

and a change of system

$$\nabla_{A}\chi \equiv \mathscr{A}^{\Phi}_{A}\nabla_{\Phi}\chi\,. \tag{3.2}$$

(These 2^n operators ∇_A or ∇_Φ are in general linearly independent). Now finally, to transform the collineation index to a simple spinvector index, we take the image of our vector ψ under the collineation ∇_A in two ways:

$$\Box^{a}\chi \equiv (\nabla^{a}{}_{b}\chi)\psi^{b} \equiv \psi^{\Phi a}\nabla_{\Phi}\chi\,, \qquad \overset{*}{\Box}{}_{a}\chi \equiv \psi_{b}\nabla^{b}{}_{a}\chi \equiv \psi^{\Phi}{}_{a}\nabla_{\Phi}\chi\,, \tag{3.3}$$

cf. (1.8) and (1.10).

The curvature theory of the collineation derivative ∇_A concerns the curvature $\mathscr{R}_{A}{}^{b}{}_{a}$ defined the by alternation of (even sum) p-vector covariant differentiation:

$$(p+q)!\,\nabla_{[\Pi_p}(\nabla_{\Phi_q]}\chi) \equiv -\mathscr{R}_{\Lambda_{2j}}\chi\,, \qquad (\Lambda_{2j} \equiv [\Pi_p\Phi_q]) \qquad [j = 0, 1, \ldots, \nu]$$
$$\mathscr{R}_{\Lambda_{2j+1}} = 0 \qquad [j = 1, \ldots, \nu-1]. \tag{3.4}$$

as for the spin-collineation curvature [4]. (The index A is then equivalent to two spinvector indices via the $\mathscr{A}^{\Phi}_{A}$). This can be shown to give

$$-1/(2j)!\,\mathscr{R}_{\Lambda_{2j}} \equiv \Gamma_{\Lambda_{2j}} = \Gamma_{[\lambda_1\lambda_2}\Gamma_{\lambda_3\lambda_4}\ldots\Gamma_{\lambda_{2j-1}\lambda_{2j}]}\,, \tag{3.5}$$

where the basic curvature invariant $\Gamma_{\lambda_1\lambda_2}$ is (proportional to) the ordinary curvature of our connection, $\mathscr{R}_{\lambda_1\lambda_2}$:

$$-2\Gamma_{\lambda_1\lambda_2} \equiv -2\partial_{[\lambda_1}\Gamma_{\lambda_2]} - 2\Gamma_{[\lambda_1}\Gamma_{\lambda_2]} = -\tfrac{1}{4}N_{\lambda_1\lambda_2\pi\xi}\eta^{[\pi\xi]}\,, \tag{3.6}$$

$N_{\lambda_1\lambda_2\pi}{}^{\xi}$ the curvature of $\overset{s}{\Gamma}_{\lambda\pi}{}^{\xi} = \left\{ \begin{matrix} \xi \\ \lambda\pi \end{matrix} \right\}_s$. Thus there are essentially no new differential invariants beyond $\mathcal{R}_{\lambda_1\lambda_2}$: This curvature tensor is a real tensor.

On the other hand, the curvature associated with $\Box$ and $\overset{*}{\Box}$ of (3.3) is essentially different, for it involves the derivative of ψ and $\overline{\psi}$. In fact, there are some ambiguities in defining this curvature; that is, there seem to be several natural ways of doing it, which result in differences of detail. For this reason, we will forego full generality and treat only the curvature associated with the simpler covariant derivatives ∇, $\overset{*}{\nabla}$:

$$\nabla^a \equiv \psi^{\varphi a}\nabla_\varphi + \psi^a\,, \qquad \overset{*}{\nabla}_a \equiv \psi_a^{\varphi}\nabla_\varphi + \psi_a\,, \tag{3.7}$$

for which there is no ambiguity. Note first that since

$$\nabla_{\Phi_{2j+1}}\chi = \Gamma_{[\Phi_{2j}}\nabla_{\varphi]}\,, \qquad \nabla_{\Phi_{2j}}\chi = \Gamma_{\Phi_{2j}}\chi\,,$$

that $\Box$, $\overset{*}{\Box}$ are simply linear combinations of vector and scalar derivatives, and hence the differential invariants of $\Box$, $\overset{*}{\Box}$ obtained in the « natural way » should be expected to yield nothing new beyond the curvature associated with ∇, $\overset{*}{\nabla}$. Second, note that $\Box$, $\overset{*}{\Box}$ reduce to ∇, $\overset{*}{\nabla}$ resp. in the case of vanishing $N_{\mu\nu\xi}{}^{\lambda}$, in which case all the ∇_{Φ_p} $(2 \leqq p \leqq n)$ become zero operators. Third, the curvature of ∇, $\overset{*}{\nabla}$ seems to suffice for the geometrization of matter, see infra.

We define

$$(\overset{*}{\nabla}_a\nabla_b - \nabla_b\overset{*}{\nabla}_a)A^{d}_{c'} \equiv -R_{ab}{}^{d}{}_{c}A^{c}_{c'} + R_{ab}{}^{d'}{}_{c'}A^{d}_{d'} \qquad [\nabla_b \equiv g_{bc}\nabla^c] \tag{3.8}$$

where the occurrence of starred and unstarred ∇ is due to the symmetrical treatment of complex conjugates. Keeping in mind $\nabla_\varphi A^{d}_{c'} = 0$, $\nabla^2_{[\pi\varphi]}A^{d}_{c'} \equiv$ $\equiv \Gamma_{\pi\varphi}{}^{d}{}_{c}A^{c}_{c'} - A^{d}_{d'}\Gamma_{\pi\varphi}{}^{d'}{}_{c'}$, this gives as curvature

$$R_{ab} \equiv 2\psi^{\pi}{}_{a}\overline{\psi}^{\varphi}{}_{b}\Gamma_{\pi\varphi} - A(\psi^{\pi}{}_{a}\nabla_\pi\overline{\psi}_b + \nabla_\pi\psi_a\overline{\psi}^{\pi}{}_{b})\,. \tag{3.9}$$

This curvature has several remarkable properties. First, althoug it is no real tensor, its scalar will be shown to be real. Second, its scalar is a function of the curvature $N_{\mu\nu\lambda}{}^{\xi}$ only in the form of its scalar N! The first of these is essential for the geometrization of matter; the second, for the theory's tie-up with Conformal Relativity. The curvature scalar is

$$R \equiv g^{bd}R_{ab}{}^{a}{}_{d} = -2\psi^{\pi}\Gamma_{\pi\varphi}\psi^{\varphi} - \psi^{\pi}\nabla_\pi\psi + \nabla_\pi\psi\psi^{\pi}, \tag{3.10}$$

remembering the index moving rules (1.9). Expanding the first term, using (3.6) we get

$$
\begin{aligned}
(3.11)\qquad -2\psi^{\pi}\Gamma_{\pi\varphi}\psi^{\varphi} &= -\tfrac{1}{4}N_{\pi\varphi\varrho_1\varrho_2}\psi^{\pi}\{\eta^{[\varrho_1\varrho_2]}\eta^{\varphi}\}\psi \\
&= -\tfrac{1}{4}N_{\pi\varphi\varrho_1\varrho_2}\psi^{\pi}\{\eta^{[\varrho_1\varrho_2\varphi]}+2\eta^{[\varrho_1}S^{\varrho_2]\varphi}\}\psi \\
&\quad -\tfrac{1}{2}N_{\pi\varphi\varrho_1\varrho_2}S^{\varrho_2\varphi}\psi\eta^{\pi}\eta^{\varrho_1}\psi \\
&= +\tfrac{1}{2}N_{\pi\lambda}\psi\eta^{\pi}\eta^{\lambda}\psi \\
&= +\tfrac{1}{2}N_{\pi\lambda}S^{\pi\lambda}(\psi)^2 \\
&= +\tfrac{1}{2}N(\psi)^2 ,
\end{aligned}
$$

where we have used (1) and various symmetries of $N_{\mu\nu\xi}{}^{\lambda}$

$$
(3.12)\qquad N_{\pi\langle\varphi\varrho_1\varrho_2\rangle}=0\ \ (^{12}),\qquad N_{\pi\varphi(\varrho_1\varrho_2)}=0,\qquad N_{[\pi\lambda]}=0 .
$$

This first term is thus obviously real. To prove the reality of the second term $\nabla_{\pi}\psi\psi^{\pi}-\psi^{\pi}\nabla_{\pi}\psi$:

$$
(\nabla_{\pi}\psi\psi^{\pi})=\nabla_{\pi}\overline{\psi}\,\overline{\psi}^{\pi}=-\nabla_{\pi}\overline{\psi}\bar{g}^{1}\psi^{\pi}=-\psi^{\pi}\nabla_{\pi}(\widetilde{\bar{g}^{1}}\,\overline{\psi})=-\psi^{\pi}\nabla_{\pi}(\bar{g}^{1}\,\overline{\psi})=-\psi^{\pi}\nabla_{\pi}\psi ,
$$

which is the second part of the second term, etc. Thus R is real, as maintained.

4. – Spinor Relativity.

Consider the unified field theory (a « relativity » in the sense of [1]) obtained by requiring that the total (i.e., integrated) spin curvature of ∇ and $\overset{*}{\nabla}$ be a minimum:

$$
(4.1)\qquad \delta\int R\sqrt{S}\,(\mathrm{d}x)=0 ,\qquad [(\mathrm{d}x)=\mathrm{d}x^0\,\mathrm{d}x^1\ldots\mathrm{d}x^5]
$$

$$
R=\tfrac{1}{2}N(\psi)^2+\nabla_{\pi}\psi\psi^{\pi}-\psi^{\pi}\nabla_{\pi}\psi .
$$

This condition is all that shall be required of the geometry.

Choosing a canonical frame $\{(\pi'),(a')\}$ (in the strict sense of section 2), then $\eta^{\pi'}$ and g are constants; hence we see that the independent fields in

(12) $\langle\ \rangle$ around a set of indices indicates the sum of cyclically permuted terms.

$R\sqrt{S}$ can be taken to be $A^{\pi}_{\pi'}$((π) any holonomic system) and ψ. $A^{\pi}_{\pi'}$ in fact occurs in N, $\sqrt{S}$, and in the spin connection in the derivatives $\nabla_{\pi}\psi$ in the simple combination $S^{\pi'\xi'}A^{\pi}_{\pi'}A^{\xi}_{\xi'} = S^{\pi\xi}$ ($S^{\pi'\xi'}$ constants); but it is noteworthy that the last two terms of R, which can be written $(\nabla_{\pi}\psi\eta^{\pi'}\psi - \psi\eta^{\pi'}\nabla_{\pi}\psi)A^{\pi}_{\pi'}$ are sensitive also to variations of $A^{\pi}_{\pi'}$ which leave $S^{\pi\xi}$ fixed. If one now varies the $A^{\pi}_{\pi'}$ and then multiplies the resulting field equation by $S_{\pi'\xi}$, one gets the following equivalent pair of equations:

$$(4.2)\quad a. \qquad (\psi)^2 G_{\pi\xi} + T_{\pi\xi} + \nabla_{(\pi}M_{\xi)} - S_{\pi\xi}\nabla_{\lambda}M^{\lambda} + \nabla^2_{\pi\xi}(\psi)^2 - S_{\pi\xi}\overset{2}{\nabla}{}^{\lambda}_{\lambda}(\psi)^2 = 0\,,$$

$$b. \qquad \nabla_{[\pi}\psi\psi_{\xi]} - \psi\nabla_{[\pi}\psi_{\xi]} = 0\,,$$

where $G_{\pi\xi} = N_{\pi\xi} - \frac{1}{2}S_{\pi\xi}N$ is the Einstein tensor function of $N_{\pi\xi}$, M^{λ} is defined (1.12), $\overset{2}{\nabla}{}^{\lambda}_{\lambda} = S^{\lambda\xi}\nabla^2_{\xi\lambda}$ is the Laplacian, and the symmetric tensor $T_{\pi\xi}$ is defined

$$(4.2)\quad c. \qquad T_{\pi\xi} \equiv \nabla_{(\pi}\psi\eta_{\xi)}\psi - \psi\eta_{(\xi}\nabla_{\pi)}\psi - S_{\pi\xi}(\nabla_{\pi}\psi\eta^{\pi}\psi - \psi\eta^{\pi}\nabla_{\pi}\psi)\,.$$

The variation wrt ψ_a gives immediately

$$(4.3)\quad a. \qquad \eta^{\pi}\nabla_{\pi}\psi - \tfrac{1}{4}N\psi = 0\,.$$

Since R is real, the variation with respect to ψ^a gives simply the complex conjugate of (4.3)a.:

$$(4.3)\quad b. \qquad \eta^{\pi}\nabla_{\pi}\overline{\psi} - \tfrac{1}{4}N\overline{\psi} = 0\,.$$

(4.2) and (4.3) are the field equations of Spinor Relativity.

Remarks: We note without proof various important features:

1) Spinor Relativity reveals itself a generalization of Conformal Relativity

$$G_{\pi\xi} = 0\,,$$

which describes in addition the field ψ. Conformal Relativity described gravitation, electromagnetism, and several mesons in the framework of the conformal physical geometry. Spinor Relativity adds matter to this scheme without altering the physical geometry (see remark 3.). The various com-

ponents of the symmetric tensor $T_{\pi\xi}$ comprise not only a matter energy tensor but also currents of Dirac form (this is seen best in a suitable non-holonomic system ([13])). The other terms in (4.2)*a.* and the equations (4.2)*b.* are unexpected from the classical viewpoint. The interpretation of these fields (classical, quantum, ...) is left open, but (at least) one interpretation must do for all.

With (4.3) is demonstrated the derivation of a Dirac equation from pure curvature; as a mass term one gets the curvature invariant of Conformal Relativity! Although the physical interpretation of ψ in this theory is far from settled, we hope that here for the first time a unified field theory can dispense with the representation of matter as singularities of the field equations, which has heretofore been necessary. This theory then can say something about the « fine structure » of matter. Contact with physical experience will now be made by making a weak field approximation and solving the combined set of field equations for the first few of the successive approximations, preferably until non-linear effects begin to make themselves felt.

2) In contrast to the Dirac electron theory, we have to do with the conformal instead of metric (« Lorentz ») geometry as physical geometry, hence ([14]) with an eight- rather than four-dimensional spin space. Preliminary calculations already have shown that this annihilates the group theoretical difficulty of indefinite sign of free particle energy, which made the ordinary Dirac theory untenable without quantization. Moreover, other reasons apart, an S_8 will be required to display particle and anti-particle in their true light as parts of one (geometrical) entity.

3) *The Principal Theorem of Spinor Relativity* ([15]):

The physical and automorphic geometries coincide, and are none other than the fourdimensional flat conformal geometry C_4 of signature $(+++-)$1.

Proof: If H_I [$I = \pi\xi$ or a] represents the fixed differential expressions defining these field equations, then *because* H_I *is also a set of tensors* under an allowable transformation $A^I_{I'}$ (non-singular affine transformation in local S_8's, non-singular real projective transformation in local P_5's) we have

$$H_{I'}(\eta^{\varrho'\alpha'}{}_{b'}) = A^I_{I'} H_I(\eta^{\varrho\alpha}{}_b) = 0\,, \tag{4.4}$$

([13]) E.g., a system introducing the five inhomogeneous manifold coordinates ξ^α ($\alpha = 1, ..., 5$). In [1] was used $A^0_\varrho = -x_\varrho$; $A^\alpha_\varrho = \partial_\varrho \xi^\alpha$, ($\varrho$) holonomic.

([14]) Cf. [1], section VII.

([15]) Cf. [1], section VII.

where the dependence on the η^{ϱ} has been explicitly indicated. An *automorphism* $P^{I}_{I'}\{P^{a}_{b'}, P^{\mu}_{\varrho'}\}$ is defined to be any allowable transformation which leaves the $\eta^{\varrho a}{}_{b}$ numerically fixed

$$\eta^{\varrho' a'}{}_{b'} = \eta^{\varrho a}{}_{b} \qquad [\varrho' = \varrho;\ a', b' = a, b \text{ resp.}]. \tag{4.5}$$

For, under an automorphism, comparing (4.4) and (4.5), we see that the old components of the field equations go tensorially into the new, and the η^{ϱ} figuring in them go into themselves (meaning, into the same matrices). Hence there go into themselves also $S_{\mu\varrho}$ and (up to a factor) the spin metric g. (Incidentally, $P^{a}_{b'}$ is then a *real automorphism*). In the usual language of mechanics, automorphisms define the class of *preferred systems* associated with a given system.

As is well known, (4.5) establishes a (1 - 1) isomorphism $P^{a}_{b'} \longleftrightarrow P^{\mu}_{\varrho'}$ ([16]) and hence a (1 - 1) isomorphism $P^{I}_{I'} \longleftrightarrow P^{\mu}_{\varrho'}$ between the automorphic group and the group of $P^{\mu}_{\varrho'}$. But the latter is the group of (normalized) *quadric collineations* $P^{\mu}{}_{\varrho}$ fixing the quadric

$$S_{\mu\nu} P^{\mu}{}_{\varrho} P^{\nu}{}_{\tau} = S_{\varrho\tau}\,. \qquad [P^{\mu}{}_{\varrho} \equiv P^{\mu}_{\varrho'},\ \varrho' = \varrho]. \tag{4.6}$$

The points of the local P_5 represent the hyperspheres (referred to hexaspherical coordinates Z^{μ}) of a local X_4 (whose points are the nullspheres) whose equation is

$$S_{\mu\varrho} Z^{\mu} Z^{\varrho} = 0\,.$$

Quadric collineations form the group taking each local X_4 into itself. But by Klein's theorem ([17]) this group is none other than the group of conformal transformations in each X_4. The signature $(- + + + - +)1$ of the quadric implies the signature $(+ + + -)1$ of the local conformal geometries in view of the reality of the local P_5's. Now for any relativity, the *physical group* (short for group of physically equivalent observers) is defined as that subgroup of the automorphic group inducing conformal transformations in the local X_4's this way; *it is the largest group of transformations between preferred systems which preserves angle.* But by Klein's theorem cited supra, the physical and automorphic groups coincide for this relativity (as for Conformal

([16]) Cf. Veblen and Givens: *Geometry of Complex Domains*, lectures given at the Inst. for Adv. Study (1935-36), p. 5-11. $P^{a}_{b'}$ is here viewed as a collineation.

([17]) F. Klein: *Vorlesungen über Höhere Geometrie*, 3d. ed. (Berlin), p. 199.

Relativity) and embrace the whole conformal group of the local X_4's. The geometries defined by these groups are thus identical, which was to be shown.

The principal theorem thus implies that our field equations admit the whole conformal group as automorphic group. In particular we note that this gives a conformally invariant-in-form Dirac-type theory of matter ([18]) and Einstein-type theory of gravitation. Not only is (four-dimensional) length no longer an invariant under the physical group, but we are led to believe by J. HAANTJES' work [6] that rest mass (once a posteriori defined here) will also go out as a physical invariant. The kinematical interpretation of a conformal transformation is a *uniform acceleration* between cartesian frames [5]. The *Principle of Spinor Relativity* (corresponding to the *Principle of Relativity* for Einsteinian relativity) thus says: The nature laws « look the same » (i.e., are invariant in form) for all observers in uniform relative acceleration.

4) This theory cannot be considered logically satisfying until the reason for the appearance of ψ is made clear. That is, it should be shown that the choice of the spinconnection of (3.7) is inevitable, that there are essentially no other invariants usable except its curvature, etc. The handling of spinspace as a projective space instead of « normalizing » it to an affine space as we have done for convenience, would certainly introduce new invariants. But we do not want just any new invariants, we need precisely one, a spinvector, beside the quantities of Conformal Relativity. This is a problem to which we shall return.

([18]) For an ad hoc conformally invariant electron theory, which is restricted to a *flat* conformal space and the surface of the hyperquadric representing the null spheres (« zero gauge »), see P. A. M. DIRAC: *Ann. of Math.*, **37**, 429 (1936).

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RIASSUNTO (*)

La parte puramente geometrica del lavoro porta notevoli contributi alla teoria della metrica e della connessione negli spazi spinoriali. La parte fisica consiste in una teoria unificata dei campi, tendente a comprendere anche i campi materiali, ottenuta ponendo una semplice (Einsteniana) condizione per la curvatura della connessione spinoriale adottata. In merito alla teoria che ne risulta, rileviamo in modo particolare che essa generalizza la Relatività Conforme dell'Autore, che ammette il gruppo conforme come gruppo fisico, e che la teoria della materia che ne risulta ha la forma della teoria di Dirac.

(*) *Traduzione a cura della Redazione.*

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On the universal Fermi-Type interaction (III).

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Summary. — An analysis is made of the symmetry requirements which can be imposed upon a linear combination of the odd couplings used in I and II to study the possibility of a universal Fermi-type interaction. A unique criterion results if one takes into account some well established experimental facts and makes the assumption that the universal interaction has the same form for all processes. The remaining ambiguity between the different modes still allowed by this criterion to the various processes can be removed by further assuming that some rule exists to discriminate among them. Two alternatives then result which lead to different shapes of the muon spectrum. A final decision depends therefore upon the precise knowledge of this spectrum. Both alternatives compare favorably for β-decay with all the experimental evidence thus far.

1. – In studying the possibility of a universal Fermi interaction [1-4] among any four of the five fermions — neutron (N), proton (P), muon (μ), electron (e), neutrino (ν) — two main questions arise: 1) that of obtaining selection rules which forbid the unobserved processes; 2) that of determining the form of the interaction for all occurring processes. An additional postulate is that such form be the same for all processes; although unrelated to a large extent

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(1) C. Yang and J. Tiomno: *Phys. Rev.*, **79**, 495 (1950).

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to the existence of selection rules, this assumption appears as a reasonable one, and is of great help in treating 2). It will be discussed later in greater detail; we begin by ignoring it.

The first point has already been considered in I, II, A. It was shown there that the wanted selection rules result if the behavior of the spinor fields under improper Lorentz transformation is suitable prescribed. Each field is attributed an individual phase factor — type — under such transformations [5] so that only the observed processes are allowed. Whether this is merely an expedient which happens to work well, or the expression of deep-seated properties of the elementary particles, we cannot say at present. It is well known that the introduction of phase factors is equivalent to the imposition of conservation laws, such as conservation of nucleons; the converse approach, i.e. to start from assumed conservation properties to deduce the universal interaction has the shortcoming that we may well ignore some of the conservation laws obeyed by the fermions. If we simply ask for Lorentz invariance and fit the types to secure it, the resulting conservation properties can be checked against our knowledge of elementary phenomena: the results obtained this way in A appear quite satisfactory.

We propose here to investigate the second question in some detail in order to determine how far symmetry arguments can help in fixing the form of the interaction. A tentative answer was already given in II, where it was shown that a particular criterion — that of antisymmetrizing pairs of particles — proved satisfactory. We shall again find this to be the case, although other possibilities exist which should not be altogether overlooked.

2. – The processes which result allowed are:

$$\text{(I)} \qquad \begin{cases} N \to P + e + \bar{\nu} \\ \mu + P \to N + \nu \\ \mu \to e + \nu + \nu \quad (\text{or: } \mu \to e + \bar{\nu} + \bar{\nu}) \end{cases}$$

and

$$\text{(II)} \qquad \text{particle} + \text{antiparticle} \to \nu \to \nu \quad (\text{or: } \bar{\nu} + \bar{\nu}) \,.$$

We must take for this [4] as Hamiltonian density a linear combination of the

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[5] This clearly implies the reduction of the quantum-mechanical ray space to vector space, as in usually done in defining boson parities. The concept of type, as considered in ref. (1) and A (see also: G. Racah: *Nuovo Cimento*, **14**, 322 (1937), where it was formulated for the first time) is only the straightforward extension of that of parity to the case of fermions.

pure odd couplings (II - 13):

$$(1)\qquad \left|\begin{array}{l} J = \langle a\gamma^4 c\rangle\langle bi\gamma^4\gamma^5 d\rangle \\ J' = \langle ai\gamma^4\gamma^5 c\rangle\langle b\gamma^4 d\rangle \\ J'' = \langle ai\gamma^k\gamma^5 c\rangle\langle b\gamma^k d\rangle + \langle a\gamma^k c\rangle\langle bi\gamma^k\gamma^5 d\rangle \\ J_1 = \langle ai\gamma^4\gamma^k c\rangle\langle bi\gamma^4\gamma^k\gamma^5 d\rangle - \langle a1c\rangle\langle b\gamma^5 d\rangle \\ J_1' = \langle ai\gamma^4\gamma^k\gamma^5 c\rangle\langle bi\gamma^4\gamma^k d\rangle - \langle a\gamma^5 c\rangle\langle b1d\rangle\,. \end{array}\right.$$

H is given therefore by:

$$(2)\qquad H = gJ + g'J' + g''J'' + ig_1 J_1 + ig_1' J_1' + \text{Herm. Conj.}.$$

Invariance under time reversal ([6]), or charge conjugation, permits to show that the coupling constants g, as written in (2), must be real. The proof is left to the reader. When the order in which the fields are written in the Hamiltonian is changed, the pure couplings (1) undergo a linear transformation, according to the formulae (II - 14) (we may regard a, b, c, d, as labels specifying the position of the fields in H). It is advantageous, therefore, to consider (1) as the components of a vector $\boldsymbol{J}$ spanning a five dimensional space S_5 ([7]). The substitutions of the group π_4 are then matrices, and a change of the order of the particles in H is interpreted as a change of the reference frame in S_5. The vector $\boldsymbol{J}$ itself remains therefore invariant, as it is intrinsically connected to the space S_5; H can be written shortly:

$$(3)\qquad H = \boldsymbol{g}\cdot\boldsymbol{J} + \text{Herm. Conj.}.$$

To calculate a given process one needs to know: 1) the order in which the fields are written into the Hamiltonian, 2) the values of the g's for that ordering. The invariance of $\boldsymbol{g}\cdot\boldsymbol{J}$ permits then to express the result in terms of any other ordering, if one wishes so.

In the standard theories of beta-decay the position of the various fields in H was known a priori. Both in Fermi's early formulation, based on the analogy between the electron-neutrino field and the electromagnetic field, and

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([6]) E. Wigner: *Nach. Ges. Wiss. Gött.*, 546 (1932); J. Tiomno: *Thesis, Princ. Univ.* (1950); S. Watanabe: *Phys. Rev.*, **84**, 1008 (1951); H. A. Tolhoek and S. R. de Groot: *Phys. Rev.*, **84**, 150 (1951).

([7]) This convenient formalism has been introduced by L. Michel: *Proc. Phys. Soc.*, **63** A, 514 (1950). It has been used in II. See also ref. ([11]).

in Yukawa's conception of an intermediate mesonic field, the place of nucleons and leptons was fixed a priori. The universal interaction hypothesis stands on quite different grounds. The processes it purports to describe are treated as due to direct couplings; the only information available is that some particles transform into others. The underlying thought is that this is only a phenomenological way of treating these processes, which actually are manifestations of deep-seated properties of the elementary particles, *in primis* their structure. It is hoped, at best, that, granted the search for the interaction proves successful, its symmetry properties may shed some light on more fundamental issues; it is not excluded that concepts alike to Fermi's or Yukawa's may again appear a posteriori.

The situation, thus, is just the reverse of that previously existing. Privileged positions to some fields cannot be assigned a priori. Rather, since « any four fermions » can also mean « the same four fermions in a different order », the position of the fields in the Hamiltonian should be, at least to some extent, irrelevant.

This would certainly be the case if the interaction were antisymmetric in all four particles (we speak from now on, in keeping with the notation of I and II, the language of the *c*-no. theory. In *q*-no. theory « antisymmetrization » becomes « symmetrization », because of the assumed anticommutation of the various fields). This requirement, however, leads to a vanishing Hamiltonian. (With even couplings it yields the well known Wigner-Critchfield interaction, which seems ruled out by present evidence).

We are, therefore, led to formulate a less exacting criterion, which can be regarded as the generalization of the previous one when account is taken of the information available, which particles decay into which:

« The interaction is required to be antisymmetric in the particles which are created (destroyed) in a given process. It must be independent of the order given to the particles in the Hamiltonian », (to within a renormalization of the coupling constant, which may be regarded as irrelevant).

We shall see that this criterion does not by itself lead to a unique answer and shall discuss more stringent possible requirements. We wish to emphasize, however, that we only propose, here, to examine the possible forms that symmetry requirements may take, without giving a priori greater weight to any of them.

3. – We need for our discussion the reduction of the representation of the permutation group π_4 in S_5 into irreducible representations ([5]). With standard techniques one finds that S_5 reduces into two irreducible subspaces S_2 and S_3,

([5]) See ref. ([3]) for the case of even couplings.

spanned respectively by the vectors

$$S_2: \begin{cases} I_1 = \frac{1}{2}(J + J') + \frac{1}{\sqrt{2}} J'' \\ I_2 = \frac{1}{2}(J + J') - \frac{1}{\sqrt{2}} J'' \end{cases}$$

$$S_3: \begin{cases} I'_1 = \frac{1}{\sqrt{2}}(J - J') \\ I'_2 = \frac{i}{\sqrt{2}}(J_1 + J'_1) \\ I'_3 = \frac{i}{\sqrt{2}}(J_1 - J'_1) \end{cases}$$

The Young patterns corresponding to each representation are indicated above. For convenience of the reader we report the form to which (II - 14) now reduce:

$$(ac)_2 = (bd)_2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

$$(ad)_2 = (bc)_2 = \begin{pmatrix} \frac{7}{4\sqrt{2}} & \frac{1}{2} + \frac{5}{4\sqrt{2}} \\ \frac{1}{2} - \frac{5}{4\sqrt{2}} & -\frac{7}{4\sqrt{2}} \end{pmatrix},$$

$$(ab)_2 = (cd)_2 = \begin{pmatrix} -\frac{7}{4\sqrt{2}} & \frac{1}{2} - \frac{5}{4\sqrt{2}} \\ \frac{1}{2} + \frac{5}{4\sqrt{2}} & \frac{7}{4\sqrt{2}} \end{pmatrix};$$

$$(ac)_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad (bd)_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix},$$

$$(ad)_3 = \begin{pmatrix} 0 & -1/2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad (bc)_3 = \begin{pmatrix} 0 & 1/2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$(ab)_3 = \begin{pmatrix} 0 & 0 & -1/2 \\ 0 & -1 & 0 \\ -2 & 0 & 0 \end{pmatrix}, \qquad (cd)_3 = \begin{pmatrix} 0 & 0 & 1/2 \\ 0 & -1 & 0 \\ 2 & 0 & 0 \end{pmatrix}.$$

(3) becomes therefore

$$H = (f_1 I_1 + f_2 I_2) + (f'_1 I'_1 + f'_2 I'_2 + f'_3 I'_3) + \text{H. C.}. \tag{4}$$

We are thus led to consider separately the two Hamiltonians:

$$H_2 = \boldsymbol{f} \cdot \boldsymbol{I} + \text{H. C.} \tag{5}$$

and

$$H_3 = \boldsymbol{f}' \cdot \boldsymbol{I}' + \text{H. C.}\,, \tag{6}$$

as they do not mix under permutations. (4) is also possible a priori, but it cannot be accepted without some further restrictions on the constants (for instance, the rather well established Fierz conditions [9])

$$g_S g_V = g_A g_T = 0\,,$$

would then be violated: this follows from the relations existing between odd and even couplings, which will be recalled later).

In applying our criterion, we can envisage two different situations:

1) one fermion decays into three (say, $N \to P + e + \bar{\nu}$);

2) two fermions interact and change into two others (say, $\mu + P \to N + \nu$).

An ambiguity arises because it is clearly possible to picture any process as happening in either modality, for instance by changing a neutrino at right into an antineutrino at left. It may well be true that some processes are well described by 1) and some by 2): this would be quite compatible with our criterion. Before discussing the thing further, we need to see the results to which 1) and 2) lead.

Case 1) is treated by imposing antisymmetry in the three particles which are created (destroyed). According to the places given them, this results from applying the antisymmetrization operators $A(abc)$, etc. (See II). It is readily seen from the Young patterns that only S_3 can give a contribution; we report from completeness the form that these operators (normalized to be idempotent) take in S_3:

$$(7)\quad \left\{ \begin{array}{ll} A_3(abc) = \begin{pmatrix} -1 & 1/2 & -1/2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix}; & A_3(abd) = \begin{pmatrix} 1 & 1/2 & 1/2 \\ 2 & 1 & 1 \\ -2 & -1 & -1 \end{pmatrix}; \\ A_3(bcd) = \begin{pmatrix} 1 & -1/2 & -1/2 \\ 2 & -1 & -1 \\ -2 & 1 & 1 \end{pmatrix}; & A_3(acd) = \begin{pmatrix} -1 & -1/2 & 1/2 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{pmatrix}. \end{array} \right.$$

[9] M. Fierz: *Zeits. f. Phys.*, **104**, 553 (1937).

Case 2) yields for S_2 (the pairs which we antisymmetrize are indicated as in II):

$$(8)\quad \begin{cases} A_2(ab\,|\,cd)=\begin{pmatrix} \frac{1}{2}+\frac{7}{8\sqrt{2}} & -\frac{1}{4}+\frac{5}{8\sqrt{2}} \\ -\frac{1}{4}-\frac{5}{8\sqrt{2}} & \frac{1}{2}+\frac{7}{8\sqrt{2}} \end{pmatrix}; \\ A_2(ad\,|\,bc)=\begin{pmatrix} \frac{1}{2}-\frac{7}{8\sqrt{2}} & -\frac{1}{4}-\frac{5}{8\sqrt{2}} \\ -\frac{1}{4}+\frac{5}{8\sqrt{2}} & \frac{1}{2}+\frac{7}{8\sqrt{2}} \end{pmatrix}; \quad A_2(ac\,|\,bd)=\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \end{cases}$$

From S_3 too there is a contribution: writing the pairs as cd, ab, one finds, for instance, that the corresponding antisymmetrization operator changes $\boldsymbol{I}'$ into the vector $(0, I_2', 0)$.

We have here an additional source of ambiguity, as three modes appear possible for each process ([10]): if the particles α, β, transform into the particles γ, δ, we can write this as

$$\alpha + \beta \to \gamma + \delta\,,$$

or:

$$\alpha - \gamma \to \bar{\beta} + \delta\,,$$

or:

$$\alpha + \delta \to \bar{\beta} - \gamma\,.$$

It is apparent from (7), (8) that the antisymmetrization operators introduced above are projection operators: those operating upon three particles project any vector onto a same direction of S_3; those operating upon two pairs project S_5 onto a two-dimensional subspace, having one dimension in S_2 and the other in S_3. In particular, this proves that our criterion is completely fulfilled by any of the above possibilities, because directions in S_3 are intrinsic of the space and do not change if the frame of reference is changed, i.e. if a change in the ordering of the particles is performed (this can be checked, of course, by direct computation).

([10]) See also J. Tiomno and J. A. Wheeler: *Rev. Mod. Phys.*, **21**, 144 (1949).

The order being irrelevant, we fix it once for all to be the standard one for neutron decay ($a \equiv \overline{P}$, $b \equiv \bar{e}$, $c \equiv N$, $d \equiv \nu$) and $a \equiv \nu$, $b \equiv \nu$, $c \equiv \mu$, $d \equiv \bar{e}$ for muon decay. A permutation will leave H invariant, but not, of course, its separate terms. We find, for neutron decay:

Case 1):

$$H = g(J - J' + iJ_1) + \text{H. C.}. \tag{9}$$

Case 2): we must now distinguish among the three possible modes of pairing the particles:

$$N\overline{P}\,|\,\bar{e}\nu \,:\, A(ac\,|\,bd) \tag{10}$$

gives:

$$H = g(J + J') + f(J - J') + \text{H. C.}.$$

$$N\nu\,|\,\overline{P}\bar{e} \,:\, A(ab\,|\,cd) \tag{11}$$

gives:

$$H = g(J + J' + J'') + if(J_1 + J_1') + \text{H. C.}.$$

$$N\bar{e}\,|\,\overline{P}\nu \,:\, A(ad\,|\,bc) \tag{12}$$

gives:

$$H = g(J + J' - J'') + if(J_1 - J_1') + \text{H. C.}.$$

The first term in 2*a*), 2*b*), 2*c*) belongs to S_2; the second to S_3 and disappears if one assumes that S_5 is to be reduced a priori to S_2 (i.e. (4) to (5)), or, equivalently, if one imposes additional symmetry requirements on particles belonging to different pairs, as suggested by the Young patterns, or by simply postulating that the coupling constants must be real.

4. – To discuss the situation, we consider in particular the neutron decay and use Ferroni's formulae [11] which reduce any calculation performed for it with odd couplings into a numerically equivalent one with even couplings (provided the neutrino mass is zero, as we assume). We use for these the standard notation S, P, etc., reminding that we take them with Hermitean operators between the fields. One finds:

Case 1). The Hamiltonian (9) gives the same results as the even Hamiltonian:

$$H = g(S + P + V) + \text{H. C.}. \tag{13}$$

[11] S. Ferroni: *Nuovo Cimento*, **9**, 1103 (1952). Similar results, with different notation, were given also by L. Michel: *Journ. de Phys. et Rad.*, **12**, 793 (1951).

Case 2). (10), (11), (12) are respectively equivalent to:

(14) $2a)$ $H = g(S-P)+f(S+P) + \text{H. C.},$

(15) $2b)$ $H = g(S-P-T)+f(V+A)+\text{H. C.},$

(16) $2c)$ $H = g(S-P+T)+f(V-A)+\text{H. C.}.$

Experimental evidence rules out (13) and imposes that $f = 0$ in (15) and (16); (14) is also ruled out because it would not yield Gamow-Teller selection rules.

Although there is no a priori reason to discard 1) or 2) for processes other than neutron decay, we introduce now the « additional postulate » mentioned in sect. 1), i.e. that the interaction has the same form for *all* processes. The interaction, if it is to be determined by such symmetry requirements, can only derive, then, from antisymmetrization of pairs imposed to S_2 alone. One finds thus that the interaction is uniquely determined in form and has a unique coupling constant. The residual ambiguity is that left in the choice of the modes: any criterion discriminating among these must forbid $2a)$ for neutron decay.

For muon decay only two modes are possible:

$\alpha)$ $\mu\bar{e}\,|\,\nu\nu$ (or $\bar{\nu}$)

$\beta)$ $\mu\bar{\nu}\,|\,\bar{e}\bar{\nu}$ (or ν).

The first is the preferred one, if we choose as criterion the following: $A)$ « The total charge of each pair is zero ».

Then $2b)$ is also uniquely selected (for muon capture one has only to replace e with μ in $2b)$. This is the interaction proposed in II on the basis of intuitive arguments. An alternative formulation of $A)$ is:

$A')$ « The interaction Hamiltonian must be antisymmetric in all charged and all neutral particles separately» [12].

The second is preferred if we admit that:

$B)$ « Each pair contains a charged and a neutral particle, and no more than one nucleon ».

This selects $2c)$ uniquely. Alternative equivalent formulations are, of course, possible.

If the postulates which permit to narrow down the problem to this alternative should happen to be all correct, then the choice between $A)$ and $B)$

[12] A similar criterion, for even couplings, has been proposed independently by D. L. Pursey: *Proc. Int. Conf.*, cited in ref. [4].

[13] This rectifies a regrettable oversight in II.

depends upon the precise determination of the muon spectrum, in particular of its end point. α) leads indeed to a spectrum ([4])

$$\mathrm{d}W = g^2 \frac{m_e^4 m_\mu c^4}{4\pi^3 \hbar^7} (\varepsilon_0 - \varepsilon) k^2 \mathrm{d}k ,$$

β) to a spectrum ([13])

$$\mathrm{d}W = g^2 \frac{m_e^4 m_\mu c^4}{4\pi^3 \hbar^7} \left[\frac{1}{2} (\varepsilon_0 - \varepsilon) + \frac{k^2}{6\varepsilon} \right] k^2 \mathrm{d}k .$$

In the first the end point is vanishing, in the second it is not.

5. – Resuming:

Interactions other than the Wigner-Critchfield interaction are possible, which obey analogous criteria of symmetry (we consider only odd couplings).

The first (case 1) leads to negative results for neutron decay.

The second (case 2) leads for this decay to correct answers only for the modes 2*b*) and 2*c*).

The answer for muon decay will be given by the muon spectrum, when known exactly.

If one believes in the existence of a criterion permitting to decide a priori which is the mode of decay, there are then two alternative possible formulations of it, *A*) and *B*), and the knowledge of the muon spectrum will permit to select one of them.

One may remark that *A*) is particularly suggestive ([14]): it would seem to indicate a connection between the electric charge e and the constant g, a decay process appearing somehow as the combination of two fermions of opposite charge, which melt together and re-emerge as neutral particles (such as muon capture), or viceversa. If formulated as in *A'*), it would have a broader range: it would forbid, indeed, by itself all processes involving four charged or four neutral particles (and, therefore, two of the processes (II)). *It is an open question whether, then, assignments of types under only space reversal or the imposition of some conservation rule would suffice to yield all the wanted selection rules.* We do not pursue the matter further because no information would derive on the form of the interaction.

In conclusion, it is a pleasure to thank Dr. L. Michel for most interesting discussions.

([14]) « This fact (that the coupling constants have the same value in all processes (I), within experimental error) ... perhaps may be correlated to the somewhat similar fact that the electric charges of all the elementary particles are equal ». (E. Fermi: *Elementary particles*, (New Haven, 1951)).

RIASSUNTO

Vengono studiati i criteri di simmetria che possono a priori imporsi ad una combinazione lineare degli accoppiamenti dispari usati in I e II nello studio dell'interazione universale alla Fermi. Tenendo conto di alcuni fatti sperimentali bene accertati e facendo l'ipotesi che la forma dell'interazione universale debba essere la stessa in ogni caso, un solo criterio risulta accettabile. La residua ambiguità, causata dal fatto che questo criterio ancora permette varie modalità per uno stesso processo, può venire rimossa — assumendo ancora che ciò sia in principio legittimo — per mezzo di due diverse regole, ciascuna risultante in una diversa forma dello spettro di decadimento del muone. La conoscenza precisa di questo permetterà quindi di raggiungere una decisione. Ambo le regole conducono nella teoria della radioattività beta a risultati in eccellente accordo con i dati sperimentali noti a tutt'oggi.

Nuclear signals dependence on the radiofrequency field and paramagnetic catalyst.

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(ricevuto il 20 Novembre 1952)

Summary. — An apparatus for nuclear signals amplitude measurements is described. With this arrangement measurements of absorption signals amplitude from protons contained in aqueous solutions of ferric nitrate at various concentrations and various radiofrequency field values have been carried out (fig. 3). In order to explain the results obtained, an approximate theory on nuclear magnetic resonance is developed assuming that the surplus of spins in the lower states is uniformly distributed over a certain frequency range. The theory takes into account the « memory » effects that occur when the thermal relaxation time is of the order of the sweep period. In this way a general formula is obtained which gives the energy absorbed during a passage. The experimental results agree satisfactorily with this theory (fig. 6, 7, 8, 9). The comparison of experimental results with theory enables one to determine relaxation times over a certain concentration range (fig. 10). They agree well with the results given by other authors.

1. – Introduction.

Nuclear magnetic resonance in condensed matter was previously observed by electromagnetic effects by Purcell, Torrey and Pound (1) and by Bloch, Hansen and Packard (2). The phenomenon consists of an exchange of energy between a r.f. field and a system of nuclear spins immersed in a constant magnetic field. Beside the constant and r.f. fields, spin-lattice and spin-spin

(1) E. M. Purcell, H. C. Torrey and R. V. Pound: *Phys. Rev.*, **69**, 37 (1946).
(2) F. Bloch, W. W. Hansen and M. Packard: *Phys. Rev.*, **70**, 474 (1946).

interactions occur. The first are responsible for the energy transfer from nuclear spins to heat reservoir, which occurs with a characteristic time constant T_1 called « thermal » or « longitudinal » relaxation time. Both spin-lattice and spin-spin interactions determine a broadening of nuclear magnetic levels, which is inversely proportional to the « transversal » relaxation time T_2 of the Bloch theory [3] [4] [5] [6]. R.f. field alters the Boltzmann distribution of the nuclear magnetic levels tending to make the surplus number zero; on the other hand, the spin-lattice interactions tend to restore the thermal equilibrium. Therefore the amplitude of the nuclear signals depends on r.f. field, on relaxation times and, by the Curie formula, on nuclear spin. Measurements of amplitude of nuclear signals are therefore interesting in order to determine the nuclear spin and also to obtain information on the spin-lattice and spin-spin interactions [7]. These interactions can be greatly influenced by the presence of paramagnetic ions. They act indeed essentially as catalyst, since, because of their relatively strong moments, local fields arise, which broaden the nuclear magnetic levels and greatly facilitate the thermal relaxation [1] [4] [8].

With the present work, we are investigating the dependence of the amplitude of the nuclear signals on r.f. field, and catalyst concentration for protons in water. The theory of the nuclear magnetic resonance has been developed only for some limit cases in which however most of our measurements are not included [3] [1] [9] [10]. We have therefore developed an approximate general theory by which the results are satisfactorily interpreted. By comparison with the experimental results it has been also possible to obtain some information on the relaxation times. Measurements of T_1 have so far been carried out by several authors [11] by the « saturation method » which was first employed by Bloembergen, Purcell and Pound. Measurements of nuclear signals amplitude are a new method suitable for obtaining the T_1 value especially when it is on the order of the sweep period.

[3] F. Bloch: *Phys. Rev.*, **70**, 460 (1946).
[4] N. Bloembergen, E. M. Purcell and R. V. Pound: *Phys. Rev.*, **73**, 679 (1948).
[5] N. Bloembergen: *Nuclear Magnetic Relaxation* (The Hague, 1948).
[6] J. H. Van Vleck: *Phys. Rev.*, **73**, 679 (1948).
[7] G. Chiarotti e L. Giulotto: *Nuovo Cimento*, **8**, 595 (1951).
[8] N. Bloembergen: *Physica*, **15**, 386 (1949).
[9] B. A. Jacobsohn and R. K. Wangsness: *Phys. Rev.*, **73**, 942 (1948).
[10] F. Bloch and R. K. Wangsness: *Phys. Rev.*, **78**, 82 (1950).
[11] E. H. Turner, A. M. Sachs and E. M. Purcell: *Phys. Rev.*, **76**, 465 (1949); **76**, 466 (1949); N. J. Poulis: *Physica*, **16**, 373 (1950).

2. – Experimental arrangement.

The comparison between the voltages induced by different nuclei contained in the same sample is relatively easy and can be made by keeping the apparatus in the same conditions and varying the constant magnetic field. The ratio between the amplitude of the signals thus observed on the oscillograph screen gives then directly the ratio between the voltages induced by the various nuclei. By this ratio we can easily deduce the ratio between the nuclear spins ([12]) ([13]).

In our case however we had to compare the voltages induced by the protons in different samples and for various values of the r.f. field. Therefore we planned

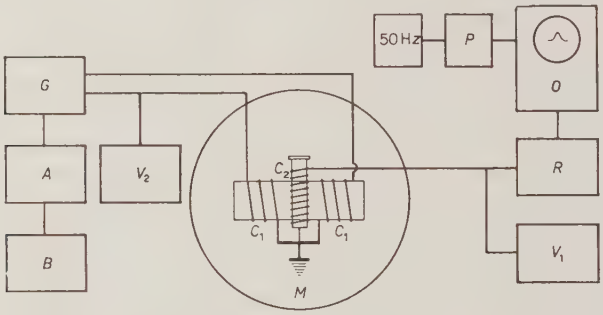

Fig. 1. – Block diagram of apparatus. G: r.f. generator; R: receiver; C_1: transmitting coils; C_2: receiving coil; M: magnet; O: oscillograph; P: phase shifter; B: beat oscillator; A: modulation amplifier.

a method by which we could compare the amplitudes of nuclear signals, and which is independent of the different conditions of the apparatus.

The arrangement for measurements of amplitude of the nuclear signals is schematically represented in fig. 1. For the observation of nuclear signals we have used the method of nuclear induction. The inductor consists of the r.f. generator G, transmitting coils C_1 and receiving coil C_2 receiver R and c.r. oscillograph O. The r.f. generator consists of an electron coupled oscillator and two amplifier stages. The receiver consists of two h.f. stages, detector and audio amplifier stages. Frequency normally used was 6,98 MHz. The

([12]) F. Bloch, A. C. Graves, M. Packard and R. W. Spence: *Phys. Rev.*, **71**, 373 (1947).

([13]) F. Alder and F. C. Yu: *Phys. Rev.*, **81**, 1067 (1951).

([14]) M. E. Rose: *Phys. Rev.*, **53**, 715 (1938).

constant magnetic field is supplied by a magnet M with pole faces shaped according to the prescription given by Kose [11] to improve the field homogeneity. The modulation of the field is provided by a 50 Hz a.c. filtered by a resonance filter. The horizontal sweep of the oscillograph is driven by the same filtered voltage, whose phase can be shifted by a RC bridge (P). The modulation amplitude of the constant field was usually ± 30 oersted. The receiving coil placed between the two transmitting coils is directly accessible, so that the sample may be easily replaced. A peculiar arrangement enables the control of the phase of the r.f. voltage across the receiving coil. Thus we can observe both the dispersion and absorption nuclear signals, which are respectively in phase and out of phase with the rotating field. The vacuum tube voltmeter V_1 is used to measure the r.f. voltage across the receiving coil. Another vacuum tube voltmeter V_2 measures the r.f. voltage across the transmitting coils. A modulator consisting of a beat oscillator (B) and an amplifier (A) enables one to modulate in amplitude the r.f. generator by varying the plate and screen grid voltage of the final tube.

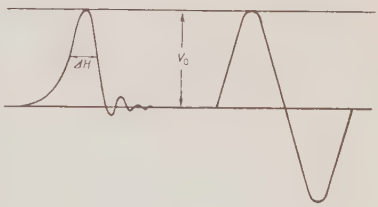

Fig. 2.

To estimate the energy absorbed by the nuclear spins system during a passage through the resonance, we measured the amplitude V_0 (in volt) and width ΔH (in oersted) of the nuclear absorption signals [15] (fig. 2).

The amplitude of nuclear signals was measured by comparison of the signals on the oscilloscope screen with a sinusoid obtained by modulating the r.f. generator. The voltage produced by nuclear magnetic resonance is .

$$V_0 = \sqrt{2}\, V_{\text{r.f.}} \cdot k \,, \tag{1}$$

where $V_{\text{r.f.}}$ is the r.f. voltage across the receiving coil, and k the modulation coefficient corresponding to the conditions of fig. 2. The comparison of the signal with a sinusoid is correct, if the receiver has a uniform response for all frequencies which occur with appreciable intensity in the Fourier spectrum of the signal.

The modulation coefficient k (of the order of 10^{-3}, 10^{-4}) was determined

[15] In our experimental conditions the signals were almost followed by « wiggles ». They follow the signal when the time passed in resonance is small, compared with the relaxation times. See N. Bloembergen, E. M. Purcell and V. R. Pound [4]; B. A. Jacobsohn and R. K. Wangsness [9]; and G. J. Béné, P. M. Denis, R. C. Extermann: *Helv. Phys. Acta*, **22**, 388 (1949).

by measuring the correspondent audio frequency input voltage (v) of amplifier A. This voltage was enhanced up to a value v', so that the corresponding modulation coefficient (k') might be measured with the peak voltmeter V_2 (fig. 1). After having verified the proportionality between modulation coefficient and modulating audio voltage, we have

$$k = k' \frac{v}{v'} .$$

The width ΔH of the signal between the half-maximum points was measured directly on the oscillograph screen by varying the constant magnetic field H_0.

With a given paramagnetic catalyst concentration we repeated the amplitude and width measurements for several r.f. field values. The r.f. field was varied by regulating the screen grid voltage of the final tube of the generator. The r.f. field value was deduced from the reading of the voltmeter V_2. We have calibrated V_2 in oersted by replacing the sample with a probe consisting of a small turn (cm² 0,62 of area) which was part of an oscillating circuit. From the measurement of the r.f. voltage in and out resonance, across the tuning condenser we can at the same time obtain the field value and the Q of the circuit. To decrease the perturbation caused by the probe it is convenient to reduce the Q of the circuit by introducing a suitable series resistor. In our case it was $Q = 4{,}2$. We observed that spurious couplings were negligible (of the order of 1%), when rotating the turn until it lies in the plane of strength lines of the field.

3. – Results.

We measured with this method the amplitude of the nuclear signals of protons in aqueous solutions of ferric nitrate ($Fe(NO_3)_2 \cdot 9H_2O$) over a large range of concentrations. We chose ferric nitrate because of its great solubility in water and because of the high value of the magnetic moment of the Fe^{+++} (about 6 Bohr magnetons).

The results are plotted in fig. 3. They give the amplitudes in volts of nuclear absorption signals in dependence on the rotating field H_1 (that is $H_{osc.}/2$) ([16]). In each curve of fig. 3 the nuclear voltage shows a maximum

([16]) As it is well known (see P. CALDIROLA: *Nuovo Cimento*, **15**, 242 (1939); F. BLOCH and A. SIEGERT: *Phys. Rev.*, **57**, 522 (1940)) the oscillating field can be effectively replaced by a rotating field of half amplitude, the rotation sense of which depends on the sign of γ.

corresponding to a certain value of H_1 and tends to zero for large values of H_1. As the concentration of paramagnetic ions decreases this maximum

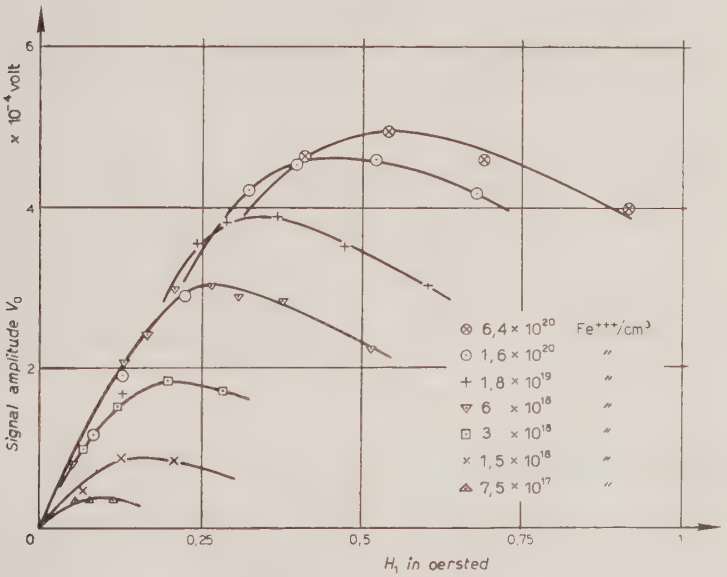

Fig. 3. – Experimental behaviour of the amplitude of protons signals V_0 as a function of rotating field H_1.

diminishes and is displaced towards weaker fields. In table I are shown the highest values of amplitude with the corresponding values of the rotating field and of width for various concentrations.

While the r.f. generator and receiver are supplied with batteries, all other instruments of fig. 1 are supplied in a.c. for practical reasons. It caused however some « noise » which made uncertain the measurement of the weaker signals. In each amplitude-determination, not less than ten instrument readings were required. Even in the most suitable cases therefore the error might amount to some percent.

The signals' width does not depend greatly upon the catalyst concentration (see table I). This result is in accordance with the JACOBSOHN and WANGSNESS' (9) theory, which is concerned however with the limit case of very weak fields.

TABLE I.

Fe^{+++}/cm^3	H_{1max} in oersted	$V_{0\,max}$ in volt $\cdot 10^{-4}$	ΔH in oersted
$6{,}4 \cdot 10^{20}$	0,58	4,95	12,2
$1{,}6 \cdot 10^{20}$	0,44	4,7	10
$1{,}8 \cdot 10^{19}$	0,35	3,9	9,5
$6 \cdot 10^{18}$	0,28	3,05	9,3
$3 \cdot 10^{18}$	0,195	1,8	9,2
$1{,}5 \cdot 10^{18}$	0,145	0,85	9,1
$7{,}5 \cdot 10^{17}$	0,09	0,4	~ 9

4. – General approximate theory.

The amplitude of the nuclear absorption signals depends on a sort of competition between the applied r.f. field and thermal relaxation. In the case of weak r.f. fields the induced magnetic moment increases proportionally to the r.f. field. For strong r.f. fields however the populations of the magnetic levels tend to equalize, and therefore the signal amplitude decreases. Thus we can explain qualitatively the behaviour of the curves of fig. 3 and also the decreasing and displacement of the maximum when the catalyst concentration decreases. This is caused by the increase of the thermal relaxation time and consequently more easy saturation.

In an exact calculation of the nuclear signals amplitude in dependence upon the r.f. field and catalyst concentration, we cannot disregard the shape of the signal; in other words it is necessary to treat the resonance problem in the more general case. This problem has been resolved only in the limit case of the « slow » and « rapid » passage, neither of which corresponds generally to our experimental conditions.

We develop here therefore a general approximate theory which gives an account of the results we have obtained.

Consider a sample containing in unit volume N nuclei of spin $\frac{1}{2}$ and magnetogyric ratio γ acted upon by a constant field H_0 and a field H_1 rotating with Larmor frequency

$$\nu = \gamma H_0 / 2\pi \,. \tag{2}$$

Let W be the probability for a stimulated transition with $\Delta m = \pm 1$ and

$$n = N_{+1/2} - N_{-1/2} , \tag{3}$$

the difference of population between the two magnetic levels, the number of transitions $\frac{1}{2} \rightarrow -\frac{1}{2}$ induced by the radiofrequency during a time $\mathrm{d}t$, is

$$n W_{1/2 \rightarrow -1/2} \mathrm{d}t ,$$

while the transitions caused by thermal relaxation are

$$(n_0 - n) \frac{1}{2T_1} \mathrm{d}t ,$$

where $n_0 = \sim N\gamma\hbar H_0/2kT$ is the surplus number at thermal equilibrium and

$$1/T_1 = W_{\text{thermal}} , \tag{4}$$

the probability that a thermal transition $-\frac{1}{2} \rightarrow \frac{1}{2}$ occurs. The surplus number resulting from the competition between r.f. field and thermal relaxation is therefore governed by the equation:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{n_0 - n}{T_1} - 2n W_{1/2 \rightarrow -1/2} , \tag{5}$$

in which the factor 2 is inserted because in one transition n changes by two.

In effect the magnetic levels are somewhat broadened because of the interaction between the nuclei and because of the limitation of mean life of a nucleus over a level. This limitation is caused by thermal agitation and reciprocal transition of spins between two nuclei. We can suppose that the surplus number n is distributed over a certain frequency range according to a normalized distribution function $g(\nu)$ ([17]).

On this assumption the probability $W_{1/2 \rightarrow -1/2}$ in dependence of frequency is

$$W_{1/2 \rightarrow -1/2} = \frac{1}{4} \gamma^2 H_1^2 \, g(\nu) . \tag{6}$$

The nuclear signals are observed usually by modulating the constant magnetic field; we suppose however that the r.f. generator frequency varies.

([17]) Note that the width of $g(\nu)$ corresponds to the signal width only in the case of « slow passage ».

In our case this frequency variation is large in respect of the width of $g(\nu)$. During a passage the transition probability depends therefore upon the time by means of $g(\nu)$. One must assume that the dependence of $g(\nu)$ on the frequency is a Gaussian (almost in the case in which particular lattice simmetries do not occur) [18]. But the integration of (5) on such assumption is practically impossibile. If, however, we use the equation (5) for determining the area of the signal, we can introduce considerable simplifications. The area of the nuclear absorption signals is indeed about independent of the shape of $g(\nu)$. We will assume therefore that $g(\nu)$ has a rectangular shape as in fig. 4, considering it constant over the whole range of the distribution which we suppose equal to $1/2T_2$. The relaxation time thus defined corresponds approximately to that introduced by BLOCH [3] and BLOEMBERGEN, PURCELL and POUND [4].

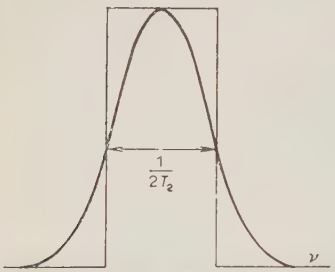

Fig. 4.

By this hypothesis and assuming that initially the surplus number is that corresponding to the thermal equilibrium, we obtain that

$$n(t) = \frac{n_0}{1 + 2W_{1/2\to -1/2}T_1}\left\{2W_{1/2\to -1/2}\exp\left[-\left(\frac{1}{T_1} + 2W_{1/2\to -1/2}\right)\right] + 1\right\}. \tag{7}$$

If τ is the time taken to pass through the resonance width, the total number of transitions induced by the radiofrequency is:

$$n_{1/2\to -1/2} = \int_0^\tau n(t) W_{1/2\to -1/2}\,\mathrm{d}t\,. \tag{8}$$

The value of τ is determined by the sweep velocity $\mathrm{d}H_0/\mathrm{d}t$ and by the width $1/2T_2$ of the distribution function:

$$\tau = \frac{\pi}{\gamma \dfrac{\mathrm{d}H_0}{\mathrm{d}t}}\,\frac{1}{T_2}\,. \tag{9}$$

The energy absorbed in a passage is

$$E = n_{1/2\to -1/2}\,\gamma\hbar H_0\,. \tag{10}$$

[18] G. E. PAKE and E. M. PURCELL: *Phys. Rev.*, **74**, 1185 (1948).

We obtain the expression of $n_{1\,2\to 1/2}$ by integrating (8) and taking into account (6) (7) and (9). If we let

$$A = \frac{\pi}{\gamma \dfrac{\mathrm{d}H_0}{\mathrm{d}t}}, \tag{11}$$

we obtain finally for the energy absorbed per unit volume in a passage:

$$E = \gamma\hbar H_0 n_0 \frac{\frac{1}{2}\gamma^2 H_1^2}{1 - \gamma^2 H_1^2 T_1 T_2} \cdot \left\{ \frac{\gamma^2 H_1^2 (T_1 T_2)^2}{1 - \gamma^2 H_1^2 T_1 T_2} \left[1 - \exp\left(- \frac{A}{T_1 T_2} (1 + \gamma^2 H_1^2 T_1 T_2) \right) \right] + 2A \right\}. \tag{12}$$

The limit case of the « slow passage » occurs when the thermal relaxation time is small in comparison with the time taken to pass through the resonance, in such a way that in each time the number of r.f. induced transitions is equal to the number of transitions caused by the thermal relaxation. At limit for $T_1 \ll \tau$ we obtain

$$E = \gamma\hbar H_0 n_0 \frac{\gamma^2 H_1^2}{1 + \gamma^2 H_1^2 T_1 T_2} A, \tag{13}$$

which coincides with the formula obtained by Bloch and by Bloembergen and coll. in the case of « slow passage ».

The opposite limit case is the « rapid passage » which occurs when the relaxation times are sufficiently long. At limit for

$$T_1 T_2 \ll 1/\gamma^2 H_1^2 \quad [19], \tag{14}$$

we obtain from (12):

$$E = \gamma\hbar H_0 \frac{n_0}{2} [1 - \exp(-A\gamma^2 H_1^2)]. \tag{15}$$

In integrating the equation (5) we have assumed that at the beginning of each passage the surplus n was that corresponding to the thermal equilibrium.

---

[19] Since we must think that $T_2 \leqslant T_1$ it is sufficient that

$$T_1 \ll 1/\gamma H_1.$$

This is true only if the time ϑ between two successive transits through resonance is sufficiently long with reference to thermal relaxation time. Otherwise we must introduce in (12) a correction factor to take into account that the Boltzmann equilibrium is not restored.

The surplus number varies in the time between two successive passages because of the thermal relaxation. If we therefore call n_2 the surplus number that occurs immediately after a passage and n_1 that we have at the beginning of the successive passage, we can write

$$\frac{n_0 - n_1}{n_0 - n_2} = \exp\left[-\frac{\vartheta}{T_1}\right]. \tag{16}$$

On the other hand comparing (10) and (15) in the case of « rapid passage », the ratio between the number of spins which undergo a r.f. stimulated transition and the surplus at the beginning of the passage is:

$$\alpha = \frac{1}{2}\left(1 - \exp\left[-A\gamma^2 H_1^2\right]\right). \tag{17}$$

After a sufficiently large number of passages we shall have therefore:

$$n_1 - n_2 = 2n_1\alpha\,. \tag{18}$$

From (16) and (18) we find finally that the surplus n_1 at the beginning of a passage is:

$$n_1 = n_0 \frac{1 - \exp\left[-\vartheta/T_1\right]}{1 - (1 - 2\alpha)\exp\left[-\vartheta/T_1\right]}\,. \tag{19}$$

This is the factor which we must substitute for n_0 in (15) to take into account this sort of « memory » of the nuclei.

We can now define in our approximation the area of the signal with

$$S = \int_0^\tau v\,\mathrm{d}t = \bar{v}\tau\,, \tag{20}$$

with $v = \chi'' H_1$ being the magnetic moment of unit volume induced by the absorption. The S so defined results proportional to the effective signal area observed on the oscillograph screen.

Considering that the immaginary part of the nuclear susceptibility is re-

lated to the absorbed power by

$$\chi'' = P/\omega H_1^2 , \tag{21}$$

we have

$$S = E/\omega H_1 , \tag{22}$$

where E is given from (12) with the correction (19), in which case it is necessary to take into account the « memory » of the nuclei.

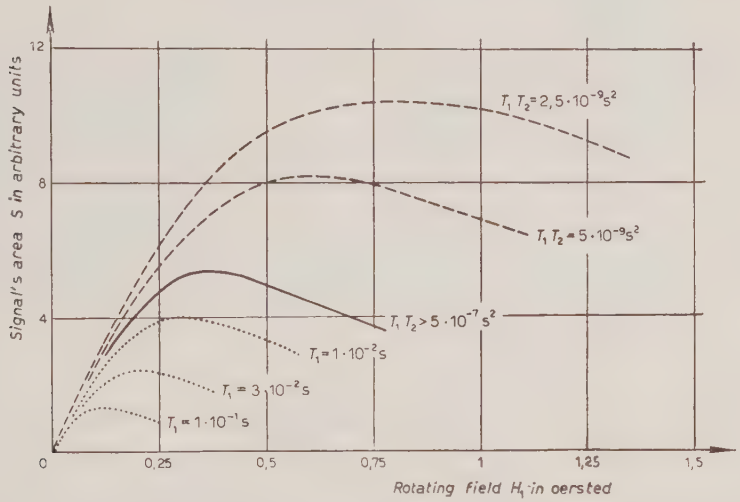

Fig. 5. – Theoretical behaviour of signals area S as a function of rotating field H_1.

In fig. 5 the area S is plotted in dependence on the rotating field H_1 for several values of T_1 and T_2 in which case $dH_0/dt = 9.5 \cdot 10^3$ oersted s^{-1} and $\theta = 10^{-2}$ s corresponding to our experimental conditions.

For the dashed lines the area S depends on the product $T_1 T_2$. If $T_1 T_2$ is sufficiently great we have the case of the « rapid passage ». The lines approach a limit curve (solid line of fig. 5) which does not depend on the product $T_1 T_2$. The dotted lines are those for which the factor (19) is appreciably smaller than n_0. Their behaviour depends only on the ratio θ/T_1.

5. – Discussion of results.

In our experimental conditions the width of the nuclear signals of absorption is not greatly dependent upon the catalyst concentration and r.f. field (see table I). Thus the areas of the signals are about proportional to the amplitude.

For this reason the behaviour of the curves of fig. 3 which give the signals' amplitude is similar to that of the theoretical curves of fig. 5 which give the areas.

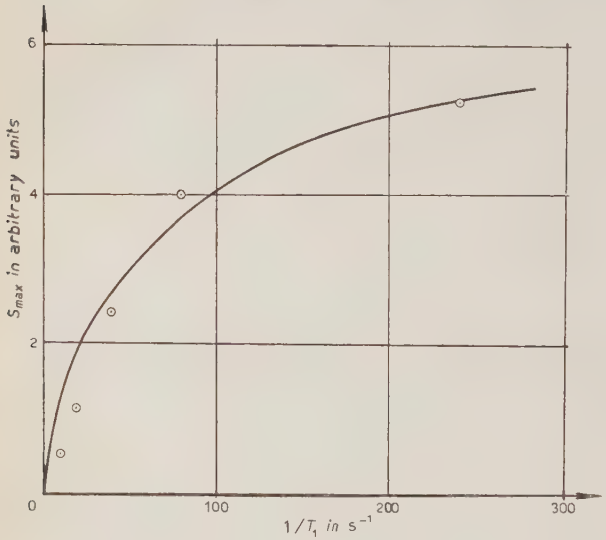

Fig. 6. – Comparison between theoretical and experimental S_{max}; in the range in which they depend only on T_1: The experimental points are plotted assuming the hypothesis that T_1 is inversely proportional to the concentration of the paramagnetic catalyst.

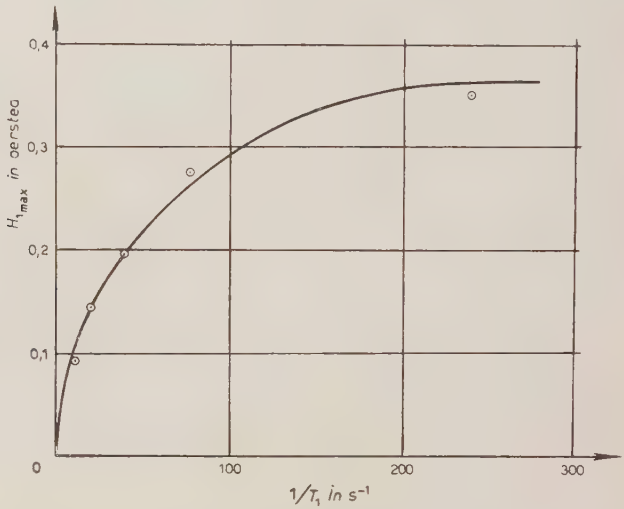

Fig. 7. – Comparison between theoretical and experimental H_{1max} in the same conditions of fig. 6.

A quantitative comparison with the theory can be made if we refer to the maximal values of the signals' areas S_{max} and to the corresponding values of the rotating field H_{1max}.

The curves of fig. 6 and 7 represent the theoretical behaviour of $S_{\max}$ and $H_{1\max}$ in dependence upon the reciprocal of the thermal relaxation time in

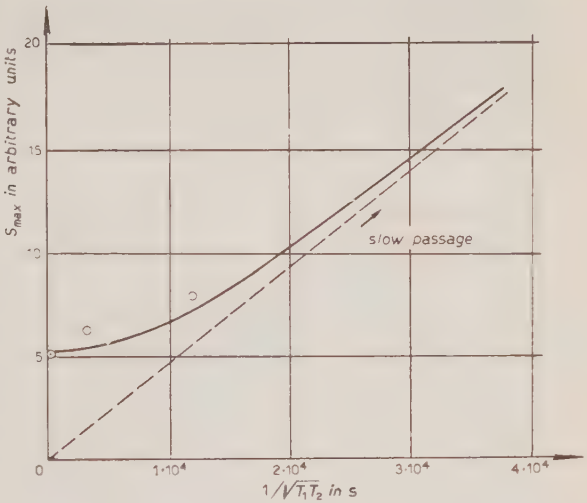

Fig. 8. – Comparison between theoretical and experimental $S_{\max}$ in the range in which they depend only on the product T_1T_2: The experimental points are plotted on the assumption that T_1 and T_2 are inversely proportional to the concentration and that T_2 is about a half of T_1.

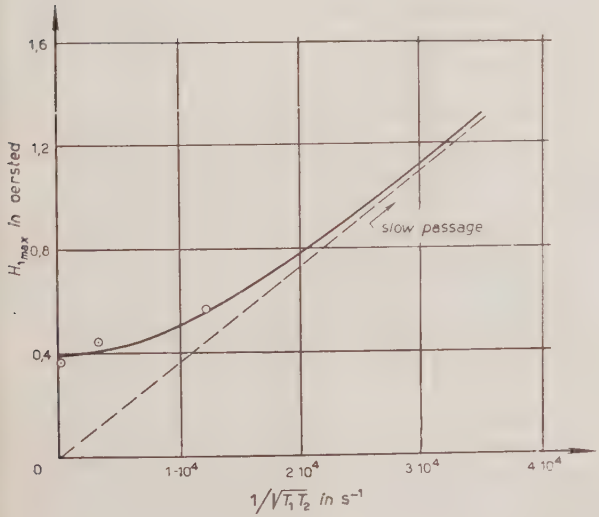

Fig. 9. – Comparison between theoretical and experimental $H_{1\max}$ in the same conditions of fig. 8.

the range in which they depend only on T_1 (dotted curves of fig. 5). The curves of fig. 8 and 9 give the theoretical behaviours of $S_{\max}$ and $H_{1\max}$ in the range in which they depend only on the product T_1T_2 (dashed curves in fig. 5).

The dashed straight line represents the limit case of the « slow passage » which the theoretical curves approach when T_1T_2 decreases.

The experimental values of S_{max} and H_{1max} have been obtained for several concentrations from the measurements of amplitude and width related in fig. 3 and table I (see Appendix).

The experimental values of S_{max} and H_{1max} relative to the smaller concentrations fit in with the curves of fig. 6 and 7 assuming the hypothesis that

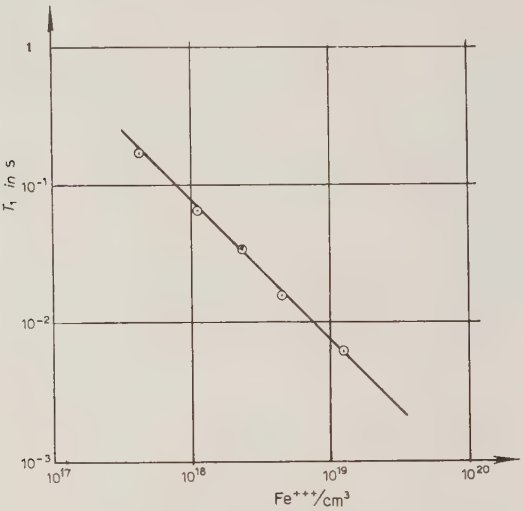

Fig. 10. – Values of T_1 versus catalyst concentration, obtained from the experimental points of fig. 7.

T_1 is inversely proportional to the concentration of the paramagnetic catalyst.

The experimental values of S_{max} and H_{1max} relative to the highest concentrations fit in with the curves of fig. 8 and 9 on the assumption that T_1 and T_2 are inversely proportional to the concentration and that T_2 is about a half of T_1.

In fig. 10 the values of T_1 in dependence upon the concentration of Fe^{+++}, obtained from the experimental points of fig. 7 are reported. These values agree well with those found by BLOEMBERGEN, PURCELL and POUND [4] and by CONGER and SELWOOD [20] using other methods. Also BLOEMBERGEN and coll. found $T_2 = \frac{1}{2}T_1$.

[20] R. L. CONGER and P. W. SELWOOD: *Journ. of Ch. Phys.*, **20**, 383 (1952).

As results from the figures, the agreement between the experimental results and our theory is in general satisfactory. However the deviations of S_{max} from the theoretical curve at smaller concentrations (fig. 6) seem to be somewhat larger than the experimental errors. This could be caused by phenomenons of hydrolisis or hydratation of the ion Fe^{+++} more pronounced at small concentrations.

Appendix

The experimental values of H_{1max} obtained by the curves of fig. 3 are directly comparable with the theoretical results, whereas the comparison between the absolute values of S_{max} is less immediate. In fact in order to obtain experimentally the value of S_{max} defined by (20) it is necessary to take into account both the shape and the dimensions of the receiving coil and of the sample, the number of turns and the quality factor of the coil and the action of the surrounding screens. The values of S_{max} related in fig. 6 and 8 are therefore determined in arbitrary units.

It is however possible to obtain with a certain approximation the absolute value of S_{max} from the experimental results. The sample having volume ϱ and section σ is placed in a coil with η turns and section σ'. If $v \cos \omega t$ is the immaginary part of the nuclear magnetisation component in the direction of the axis of the coil, the effective flux through one turn is:

$$\Phi = 4\pi\varrho(\sigma' - k\sigma)v \cos \omega t ,$$

where k is a coefficient different for the various turns which we have introduced in order to take into account the action of the superficial magnetic charges at the end of the sample and the screen surrounding the coil.

The amplitude of the induced voltage is therefore:

$$V = 4\pi \cdot 10^{-8}\, Q\omega\eta\varrho(\sigma' - \bar{k}\sigma)v ,$$

where $\bar{k}$ is the average of k.

The area of the nuclear absorption signals defined by (20) is:

$$S = \int_0^\tau v\,\mathrm{d}t = \frac{10^8}{4\pi Q\omega\eta\varrho(\sigma' - \bar{k}\sigma)} \int_0^\tau V\,\mathrm{d}t =$$

$$= \frac{10^8}{4\pi Q\omega\eta\varrho(\sigma' - \bar{k}\sigma)\dfrac{\mathrm{d}H_0}{\mathrm{d}t}} \int_{H_0'}^{H_0''} V\,\mathrm{d}H = \frac{10^8}{4\pi Q\omega\eta\varrho(\sigma' - \bar{k}\sigma)\dfrac{\mathrm{d}H_0}{\mathrm{d}t}} \cdot (V_0 \Delta H),$$

where V_0 and ΔH are respectively the signal amplitude and width measured by the method described in section 1.

The values of k has been calculated graphically (neglecting however the actions of the screens) for a cylindrical coil having 1,76 cm² section and 1,55 cm length, containing a sample having 2,35 cm³ volume and 1,5 cm² section. We have obtained $\bar{k} = 0{,}32$.

The quality factor of the coil, with the sample inside, is shown to be 44, and the frequency normally used was 6,98 MHz and the sweep velocity $dH_0/dt = 9{,}5 \cdot 10^3$ oersted s^{-1}.

We have therefore

$$S = 7.4 \cdot 10^{-9} (V_0 \Delta H) .$$

For instance, for the curve of fig. 3 of concentration $1.8 \cdot 10^{19}$ Fe^{+++}/cm^3 we have

$$S_{max} = 2.75 \cdot 10^{-11} \text{ erg s oersted}^{-1} .$$

On the other hand (22)

$$S = E/\omega H_1 ,$$

where E is given in this case by (15). We found in this way the area foreseen by the theory and obtain

$$S_{max} = 3.7 \cdot 10^{-11} \text{ erg s oersted}^{-1} .$$

The experimental value of S ($2.5 \cdot 10^{-11}$ erg s oersted^{-1}) gives a result somewhat lower than that calculated ($3.7 \cdot 10^{-11}$ erg s oersted^{-1}). This could be caused by our having neglected the action of the screens in calculating $\bar{k}$. Taking into account this action (otherwise hardly estimable) we could obtain an appreciably greater value for experimental S_{max}.

The agreement between the measured and theoretical values of S_{max} can therefore be considered satisfactory.

We wish to thank Dr. G. Lanzi for his efficient help in performing the measures. We wish also to express our thanks to the C.N.R. and the Consorzio Universitario Lombardo for their financial support which has enabled us to perform our work.

RIASSUNTO

Viene descritto un dispositivo per la misura delle ampiezze dei segnali nucleari. Sono state eseguite con questo metodo misure di ampiezza dei segnali nucleari di assorbimento dei protoni in soluzioni acquose di nitrato ferrico a diverse concentrazioni e per diversi valori del campo a radiofrequenza (fig. 3). Per interpretare i risultati ottenuti è stata svolta una teoria generale approssimata della risonanza magnetica nucleare assumendo che la differenza di popolazione fra i livelli magnetici nucleari sia distribuita uniformemente entro un certo intervallo di frequenza. Nella teoria si tien conto anche degli effetti di « memoria » che intervengono quando il tempo di rilassamento termico è dell'ordine del periodo dello scopamento. Si ottiene così una formula che dà l'energia assorbita in un passaggio e che è applicabile in ogni caso. I risultati ottenuti sono in accordo anche quantitativo con questa teoria (fig. 6. 7. 8. 9). Confrontando i risultati sperimentali con la teoria sono stati ricavati i tempi di rilassamento per un certo intervallo di concentrazioni del catalizzatore paramagnetico (fig. 10). Essi risultano in buon accordo con quelli ottenuti da altri autori.

Distribuzioni angolari dei rami neri e grigi in stelle di media ed alta energia.

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(ricevuto il 23 Novembre 1952)

Riassunto. — Sono state studiate le distribuzioni e le correlazioni angolari fra i rami neri e grigi in stelle di media (500-700 MeV) ed alta (800-4000 MeV) energia. La notevole anisotropia presente, anche per i rami meno energici, nelle stelle di media energia decresce in modo rilevante al crescere dell'energia della stella. Si notano alcune differenze nell'andamento delle distribuzioni angolari e delle correlazioni nelle stelle con e senza rinculo visibile. Il quadro fenomenologico che ne risulta viene brevemente discusso con le possibili interpretazioni.

Lo studio delle distribuzioni angolari e delle correlazioni angolari fra i rami di media e bassa energia uscenti dalle stelle di disintegrazione nelle lastre nucleari è stato oggetto della ricerca di diversi sperimentatori. Se si eccettuano però alcuni lavori nei quali si considerano esplicitamente tali distribuzioni e correlazioni solo per le stelle di pochi rami ([1,2]) e un lavoro di Harding *et al.* ([3]), per varie categorie di stelle, basato però su di una statistica assai povera, tali analisi sono state generalmente eseguite sull'insieme delle stelle, considerate come un unico complesso, senza fare distinzioni di categorie energetiche.

([1]) A. Manfredini: *Nuovo Cimento*, **8**, 194 (1951); M. Della Corte e M. Giovannozzi: *Nuovo Cimento*, **8**, 741 (1951); P. E. Hogdson: *Phil. Mag.*, **43**, 190 (1952); D. H. Perkins: *Nature*, **160**, 299 (1947); **161**, 486 (1948).

([2]) G. Lovera: *Nuovo Cimento*, **6**, 233 (1949); **9**, 134 (1952); **9**, 857 (1952).

([3]) J. B. Harding, S. Lattimore e D. H. Perkins: *Proc. Roy. Soc.*, **196**, 325 (1949).

Poichè si può oggi ritenere abbastanza probabile che il meccanismo che dà luogo alla produzione della stella possa dipendere dal valore dell'energia complessiva messa in gioco nel fenomeno stesso, ci è sembrato utile cominciare ad eseguire queste determinazioni relativamente a categorie definite di stelle, selezionate secondo criteri miranti a raggruppare in una stessa categoria stelle di eguale energia complessiva.

Riportiamo brevemente i risultati finora raggiunti in uno studio sistematico condotto sui rami neri N ($I/I^0 \geqslant 4{,}5$) e grigi G ($4{,}5 > I/I^0 \geqslant 2{,}0$) di 148 stelle di media ed alta energia.

Per ogni ramo N o G sono stati misurati: la ionizzazione specifica $I\ I^0_j$, l'angolo con la verticale o con il primario della proiezione della traccia sul piano dell'emulsione, l'angolo φ di inclinazione della traccia rispetto a questo piano; ed inoltre la lunghezza R dei N terminanti in emulsione. Per questi ultimi una discriminazione fra α e protoni, basata sul conteggio dei buchi in funzione del range R, è risultata molto incerta e del tutto impossibile per angoli di inclinazione $\varphi > 10°$. In base a tali determinazioni si è assunto per l'energia media associata ad ogni ramo, tenendo conto dei neutroni emessi, ~ 50 MeV per ogni N e ~ 150 MeV per ogni G.

Sono stati considerati « rinculi » i rami grossi con $R < 10\ \mu$.

Le stelle studiate secondo tale criterio sono:

a) Stelle di media energia: 100 stelle a 8-12 rami $N + G$ e non più di 2 rami al minimo di ionizzazione, prodotte in lastre Ilford G5 spesse 1 200 μ, esposte verticali a 4 550 m s.l.m. (Monte Rosa) (*). Tali stelle sono state selezionate in modo che la somma delle energie di tutti i rami emessi, tenendo conto quindi anche dei neutroni, cadesse tra i 500 ed i 700 MeV. Delle stelle studiate 31 hanno rinculo visibile e 69 ne sono prive.

b) Stelle di alta energia: 48 stelle a più di 13 rami N, con primario visibile e $I/I^0 < 1{,}2$, prodotte in lastre Ilford G5 spesse 400 μ, esposte verticali a 28 000 m s.l.m. (Minneapolis, U.S.A.) (**). Queste stelle sono state selezionate in modo che la somma delle energie di tutti i rami N o G, tenendo conto anche dei neutroni, cadesse per un primo gruppo (33 stelle) tra gli 800 ed i 2 000 MeV, e per un secondo gruppo (15 stelle) tra i 2 000 ed i 4 000 MeV. La identificazione dei rinculi è resa difficile e dubbia dall'elevato numero di rami e non è stato tenuto conto, nelle seguenti analisi delle distribuzioni angolari, della presenza o meno del rinculo.

(*) Gentilmente sviluppate al Centre de Physique Nucléaire di Bruxelles, con il personale interessamento del prof. Occhialini.

(**) Ottenute dall'Università di Bristol per cortesia del prof. Powell.

1. – Stelle di media energia.

Le fig. 1 e 2 danno le distribuzioni angolari $P(\vartheta)$, con ϑ angolo nello spazio rispetto alla verticale, per steradiante e ramo, dei rami N, nelle stelle con e senza rinculo visibile; la fig. 3 dà la $P(\vartheta)$ per i soli rami di energia più bassa

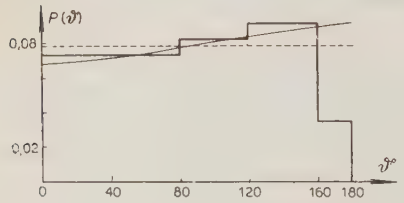

Fig. 1. – Stelle di media energia con rinculo visibile: 229 rami con $I/I^0 > 4{,}5$. $P(\vartheta) = 0{,}079 - 0{,}012 \cos\vartheta + 0{,}002 \cos^2\vartheta$.

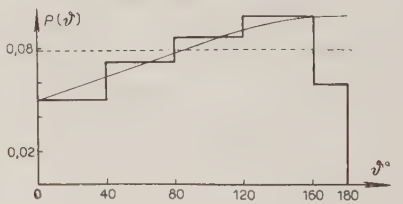

Fig. 2. – Stelle di media energia senza rinculo visibile: 551 rami con $I/I^0 > 4{,}5$. $P(\vartheta) = 0{,}082 - 0{,}025 \cos\vartheta - 0{,}007 \cos^2\vartheta$.

($I/I^0 \geqslant 6{,}5$, corrispondente a protoni di 35 MeV), appartenenti a tutte le stelle studiate.

Per le correzioni necessarie nello studio delle correlazioni angolari l'istogramma sperimentale è stato approssimato con una funzione del tipo:

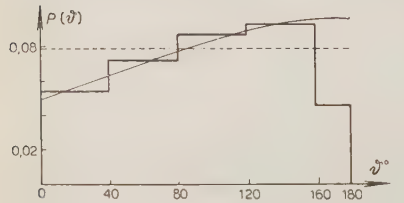

Fig. 3. – Stelle di media energia con e senza rinculo visibile: 641 rami con $I/I^0 > 6.5$. $P(\vartheta) = 0.083 - 0{,}023 \cos\vartheta - 0{,}010 \cos^2\vartheta$.

$$(1) \qquad P(\vartheta) = A + B\cos\vartheta + C\cos^2\vartheta\,,$$

con i coefficienti A, B e C opportunamente normalizzati, che è rappresentata nelle figure precedenti da una curva a tratto pieno; nelle stesse figure la retta tratteggiata indica la distribuzione isotropa. Gli angoli ϑ sono misurati a partire dalla verticale verso l'alto.

Si nota:

1) L'assenza della zona isotropa notata da BERNARDINI [1] (con gli angoli tra le proiezioni dei rami e la direzione dei primari, e con minori energie dei primari) tra 0° e 60°, anche nella $P(\vartheta)$ mostrata nella fig. 3, valida per rami di energia comparabile con quelli di BERNARDINI. La distribuzione angolare risulta al contrario notevolmente anisotropa anche a queste energie.

[1] G. BERNARDINI, E. T. BOOTH e S. J. LINDENBAUM: *Phys. Rev.*, **85**, 826 (1952).

2) Un notevole abbassamento delle $P(\vartheta)$ per grandi angoli (160°-180°): questo abbassamento è maggiore dell'errore statistico corrispondente a quel punto e non sembra spiegabile con un errore sistematico di misura.

3) Una anisotropia notevolmente più marcata nelle stelle prive di rinculo visibile rispetto a quella riscontrata nelle stelle con rinculo visibile.

Sono stati inoltre studiati gli spettri in range in due diversi angoli solidi, l'uno con $0° \leqslant \vartheta < 60°$, l'altro con $60° \leqslant \vartheta \leqslant 180°$, per mettere in luce eventuali differenze dovute al predominio nel secondo angolo solido dei rami di emissione diretta. I due spettri sono risultati, nei limiti degli errori statistici ancora troppo forti, praticamente coincidenti.

Per tale gruppo di stelle sono state anche studiate le correlazioni angolari tra i rami N, secondo i seguenti criteri: la distribuzione sperimentale degli angoli nello spazio, ψ, fra un ramo N e tutti gli altri N di una stella è stata paragonata a quella prevedibile teoricamente, $f(\psi)$, nell'ipotesi di eventi non correlati intrinsecamente e tenendo conto della distribuzione dovuta alla anisotropia delle $P(\vartheta)$. La $f(\psi)$ è data da una funzione del tipo:

$$f(\psi) = A' + B' \cos \psi + C' \cos^2 \psi \,, \tag{2}$$

con A', B' e C' coefficienti numerici che dipendono dalle A, B e C delle (1).

Le fig. 4 e 5 danno le distribuzioni osservate nelle ψ per le stelle con e

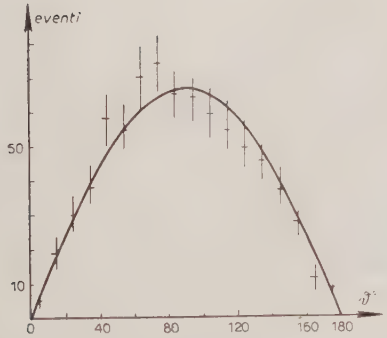

Fig. 4. – Stelle di media 'energia' con rinculo visibile: 764 angoli tra i N.

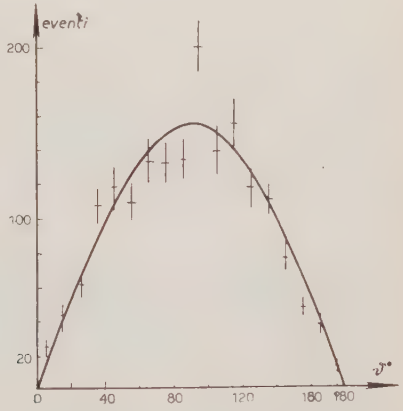

Fig. 5. – Stelle di media energia senza rinculo visibile: 1755 angoli tra i N.

senza rinculo visibile; nelle stesse figure la curva a tratto pieno indica la $f(\psi)$ data dalla (2). Si nota che nelle stelle con rinculo visibile la distribuzione sperimentale coincide, nei limiti degli errori statistici, con quella teorica, denunciando

così l'assenza di anisotropie intrinseche nelle stelle studiate. Questo dato è in accordo con quello di Lovera (²), secondo il quale l'anisotropia riscontrata nelle stelle a piccolo numero di rami va decrescendo all'aumentare di questi ultimi e diviene non apprezzabile già in stelle con 6 o 7 rami.

Invece nelle stelle senza rinculo visibile si nota un massimo marcato a 90°, notevolmente maggiore di quello prevedibile per la $f(\psi)$; la differenza tra il valore sperimentale e quello teorico è circa tre volte l'errore statistico relativo.

È stata infine studiata la distribuzione degli angoli nello spazio di ogni ramo N con $R > 300$ μ con quello successivo in ordine di range; anche questa distribuzione coincide con quella prevedibile teoricamente, $f(\psi)$, per eventi non correlati.

2. – Stelle di alta energia.

Le fig. 6 e 7 danno le distribuzioni angolari $P(\vartheta)$ per steradiante e ramo dei rami N (ϑ angolo nello spazio rispetto al primario), nelle stelle con energia

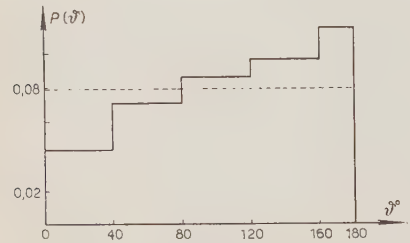

Fig. 6. – Stelle di alta energia (800-2 000 MeV): 470 rami con $I/I^0 > 4{,}5$.

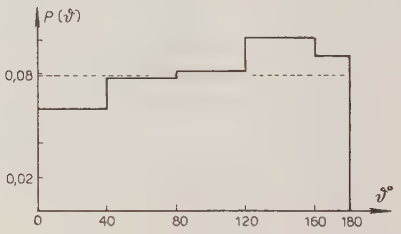

Fig. 7. – Stelle di alta energia (2 000-4 000 MeV): 309 rami con $I/I^0 > 4{,}5$.

tra 800 e 2 000 MeV (fig. 6) ed in quelle con energia tra 2 000 e 4 000 MeV (fig. 7); la fig. 8 dà la $P(\vartheta)$ per i soli rami di energia più bassa ($I/I^0 \geqslant 6{,}8$, corrispondente a protoni di 30 MeV), appartenenti a tutte le stelle studiate. Gli angoli ϑ sono misurati a partire dalla direzione del primario verso l'alto.

Si nota:

1) L'anisotropia messa in luce dalle $P(\vartheta)$ è decrescente all'aumentare dell'energia delle stelle; per il primo gruppo di stelle di alta energia, tuttavia, l'anisotropia è ancora paragonabile a quella riscontrata nelle stelle di media energia.

2) La distribuzione praticamente isotropa nella $P(\vartheta)$ dei rami di più bassa energia (fig. 8) è interrotta da un abbassamento marcato nella direzione di arrivo del primario. Anche questo abbassamento è maggiore dell'errore

statistico corrispondente a quel punto e non sembra spiegabile con un errore sistematico di misura.

La distribuzione delle G, fortemente anisotropa, non presenta alcuna caratteristica nuova e coincide con altre distribuzioni già studiate in questo laboratorio.

La distribuzione dei rami in funzione del rapporto I/I^0 sembra diversa per le stelle del gruppo più energico, nelle quali si riscontra un massimo secondario per $4{,}5 \geqslant I/I^0 \geqslant 3{,}0$, da quella osservata per le stelle del gruppo di minore energia, per le quali tale massimo non si presenta. Si cerca ora di chiarire la ragione e la natura di tale massimo secondario introducendo opportune correzioni per la presenza di deutoni e tritoni tra le traccie con ionizzazione specifica compresa nell'intervallo indicato. Un analogo massimo nello spettro I/I^0 è stato ora ritrovato anche in una serie di misure indipendenti da Ceccarelli e Zorn [5].

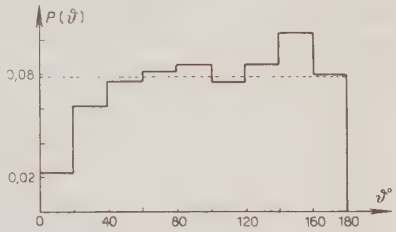

Fig. 8. – Stelle di alta energia (800-4 000 MeV): 460 rami con $I_i I^0 > 6{,}8$.

3. – Conclusioni.

Le caratteristiche riscontrate nelle distribuzioni angolari precedentemente studiate permettono alcune considerazioni di carattere preliminare sul meccanismo di formazione delle stelle studiate. Queste potranno ovviamente considerarsi confermate solo da una statistica più abbondante e più rigorosamente selezionata che è ora in corso di elaborazione.

a) *Stelle di media energia.* – Tutte le stelle di media energia studiate sono prodotte in un nucleo pesante dell'emulsione; il numero di stadi del processo di cascata iniziato dal nucleone incidente nel nucleo pesante urtato deve essere sufficientemente grande perchè si possa giustificare la mancanza di un cono con soli rami evaporativi (che porterebbe, come nel lavoro di Bernardini, ad una zona di distribuzione angolare praticamente isotropa per piccoli ϑ). La coesistenza dei rami di emissione diretta con quelli evaporativi, anche alle più basse energie, è ancora messa in evidenza dalla coincidenza degli spettri di range nei due diversi angoli solidi presi in esame e dall'abbassamento delle $P(\vartheta)$ per piccoli angoli rispetto alla verticale e quindi, approssimativamente,

[5] M. Ceccarelli e G. Zorn: In corso di pubblicazione sul *Nuovo Cimento*.

al primario. Infatti nei primi stadi del processo di cascata, durante i quali i nucleoni sono ancora strettamente collimati con il primario, le collisioni non possono dar luogo a rami di bassa energia con piccoli angoli rispetto alla direzione del nucleone incidente.

Inoltre, come si è visto, nelle stelle senza rinculo visibile la distribuzione degli angoli fra i rami neri presenta un netto massimo per angoli di 90°, corrispondenti a collisioni tra due nucleoni dopo le quali entrambi i nucleoni riescano ad uscire dal nucleo (si tratta quindi, probabilmente, di collisioni che avvengono alla superficie del nucleo e quindi appartengono ad uno stadio finale del processo di cascata). Questa interpretazione del massimo a 90° è in accordo con i recenti dati di CHAMBERLAIN e SEGRÉ ([6]) su una frequenza anomala di correlazioni a 90° fra i protoni ottenuti bombardando Li con protoni di 350 MeV, e permetterebbe di concludere che anche in un nucleo pesante e ad energia del nucleone incidente abbastanza bassa (100-150 MeV) le collisioni nell'interno del nucleo sono ancora descrivibili come una cascata di collisioni nucleone-nucleone.

Interessante è ancora l'assenza di una simile correlazione nelle stelle con rinculo visibile: questo dato, insieme all'osservazione che l'anisotropia dei rami neri in queste stelle è molto meno pronunciata che nelle altre, porta alla conclusione che nelle stelle con rinculo il contributo dell'emissione diretta è notevolmente minore che in quelle senza rinculo. Questo può essere dovuto ad una fluttuazione nel meccanismo della cascata che, a parità di energia ceduta dal primario alla stella, può avvantaggiare l'uscita dal nucleo dei termini di emissione diretta (stelle senza rinculo) o portare ad un loro notevole riassorbimento o riflessione sulla barriera (stelle con rinculo). In questo ultimo caso il rinculo sarebbe la traccia del nucleo residuo messo in moto con un impulso eguale e contrario a quello perduto nel processo evaporativo.

Alle fluttuazioni nel processo di cascata potrebbe talvolta sovrapporsi un meccanismo analogo alla fissione (PERKINS, 1948 ([1])) o alla emissione di frammenti pesanti (SÖRENSEN ([7]); CRUSSARD ([8])) che potrebbe aver luogo durante il processo di cascata, diminuendo il contributo dell'emissione diretta e quindi l'anisotropia, diminuendo pure il numero di collisioni alla superficie del nucleo e quindi il numero di correlazioni a 90°, e portando infine alla formazione di un frammento pesante di bassa energia che si presenterebbe nelle stelle sotto forma di rinculo.

b) *Stelle di alta energia.* – La diminuzione dell'anisotropia riscontrata nei rami N di queste stelle al variare dell'energia, e cioè passando dal primo al secondo gruppo, dimostrerebbe che in esse parte dei rami di bassa energia

([6]) O. CHAMBERLAIN e E. SEGRÉ: *Phys. Rev.*, **87**, 81 (1952).
([7]) S. O. SÖRENSEN: *Emission of heavy fragments...* (Oslo, 1951).
([8]) J. CRUSSARD: *Thèse de Paris* (1952).

prende origine in processi diversi dall'emissione diretta; è d'altronde difficile ammettere che tutta l'energia di eccitazione acquistata dal nucleo lo sia per semplice frenamento ed assorbimento dei nucleoni della cascata; nè a tali energie di eccitazione è accettabile un modello puramente evaporativo, che presuppone il raggiungimento di uno stato di equilibrio tra i nucleoni del nucleo composto formatosi nell'urto e il successivo iniziarsi dell'evaporazione.

Questo aumento della isotropia nelle stelle di energia più elevata rende invece plausibile l'ipotesi che, al di là di una certa energia di eccitazione, una parte notevole del fondo isotropo nella $P(\vartheta)$ dei rami N sia dovuto a rami derivanti, direttamente o indirettamente, dal riassorbimento di mesoni prodotti nelle collisioni corrispondenti ai primi stadi della cascata; produzione questa che, per tali energie, diventa certamente importante.

Ringraziamo vivamente il prof. N. Dallaporta per l'interesse con cui ha seguito tutto il nostro lavoro e per le utili discussioni durante la elaborazione dei dati sperimentali; ed il dott. A. Kind per l'aiuto gentilmente prestato nella analisi degli sviluppi matematici.

SUMMARY

The angular distributions and angular correlations of the black tracks N ($I/I^0 \geqslant 4.5$ and gray tracks G ($4.5 > I/I^0 \geqslant 2.0$) are analyzed for 148 nuclear disintegrations of low (500-700 MeV) and high (800-4000 MeV) energy, observed in Ilford G5 nuclear plates exposed to cosmic radiation. In low energy stars we note following features: the distribution of angles with respect to the vertical is considerably anisotropic even for the low energy tracks in the stars without a recoil track. The lowest energy tracks are therefore collimated in a downward direction. In the case of stars with a recoil track, the angular distribution is more isotropic. Moreover, for the stars without a recoil track the angles of 90° between the black tracks are abnormally frequent. Therefore there seems to be an intrinsic angular correlation, indipendent of the latter anisotropy. For the high energy stars, the anisotropy of the angular distribution of the black tracks seems to decrease as the energy of the stars increases: and the I/I^0 spectrum shows a secundary maximum for $4.5 \geqslant I/I^0 \geqslant 3.0$. The fenomenological picture that we have observed is discussed in relation with its possible interpretations.

NOTE TECNICHE

Generatore impulsabile di Neutroni.

E. Gatti, G. Perona e A. Persano
Laboratori CISE - Milano

(ricevuto il 7 Novembre 1952)

Riassunto. — Si descrive un impianto costituito da un acceleratore di deutoni alimentato da un generatore a 400 kV tipo Cockcroft e Walton e munito di sorgente di ioni impulsabile. L'impianto è attrezzato per la spettrografia dei neutroni lenti col metodo del tempo di volo.

1. – Generatore.

Il generatore 400 kV 3 kVA è costituito da un quadruplicatore di tensione tipo Cockcroft e Walton. Il trasformatore da 75 kV eff. è alimentato dalla rete a 50 periodi tramite un regolatore di tensione a induzione. I raddrizzatori sono diodi a vuoto spinto con i filamenti accesi mediante dinamo azionate da alberi di bakelite. L'ondulazione della tensione è di 3 kV per mA di corrente media erogata (*).

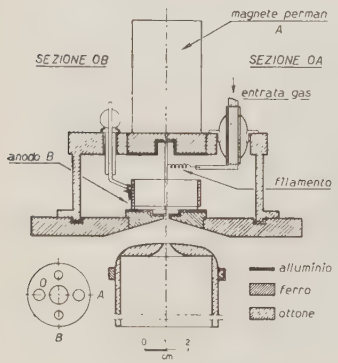

Fig. 1. – Sorgente di ioni ed elettrodo estrattore.

2. – Sorgente di ioni.

La sorgente di ioni è del tipo a campo magnetico assiale con pendolazione di elettroni [1] [2] [3]. La geometria è simile a quella impiegata da Keller [1]. La sorgente infatti è stata inizialmente costruita per funzionare con catodo freddo e ad impulsi: la successiva introduzione di un catodo caldo, che ha

(*) La costruzione e i condensatori sono della Società Passoni e Villa di Milano.

[1] R. Keller: *Helv. Phys. Ac.*, **22**, 78 (1949).
[2] J. Kistemaker e H. L. Douwes Dekker: *Physica*, **16**, 198 (1950).
[3] P. C. Veenstra e J. M. Milatz: *Helv. Phys. Ac.*, **23**, 39 (1950).

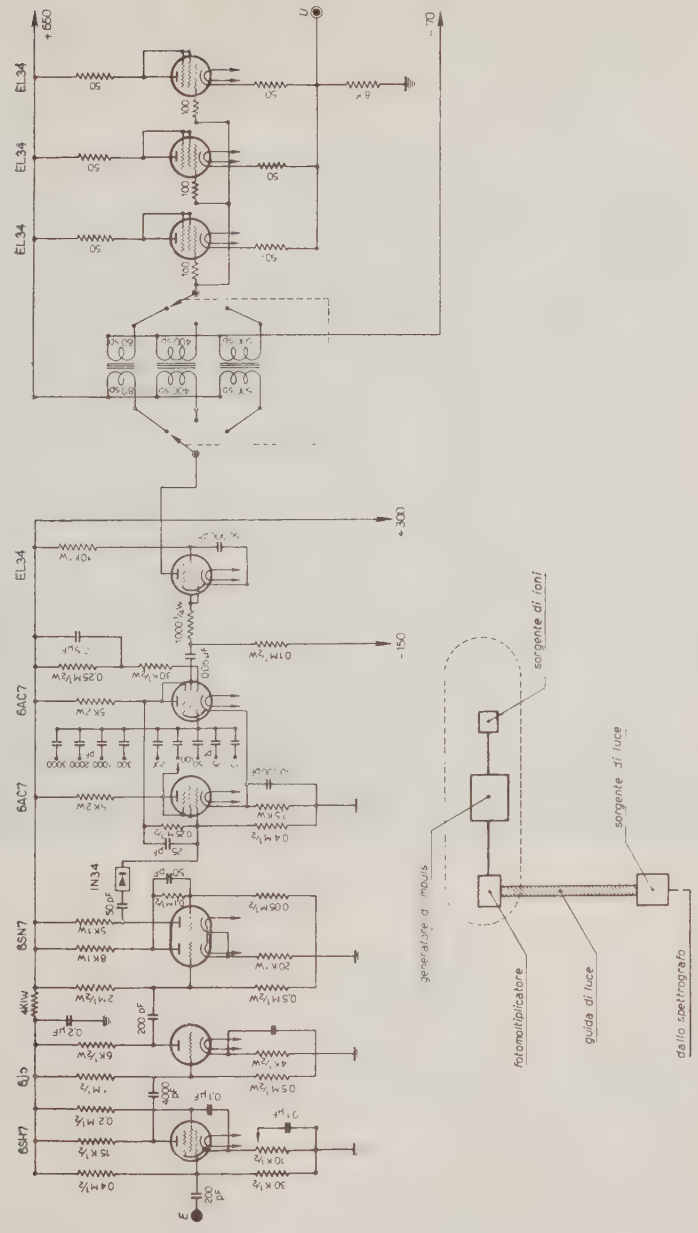

Fig. 2. – Generatore di impulsi (schema e collegamenti).

permesso di raggiungere una più precisa definizione dell'istante di accensione e un più breve tempo di salita dell'impulso (0,2 μs), permette di porla in relazione con i modelli di KISTEMAKER e di VEENSTRA di cui è noto dalla letteratura il funzionamento continuo [2] [3].

Il disegno della sorgente è dato in fig. 1. Il campo magnetico, dell'ordine di alcuni 10^{-2} weber/m^2, è fornito dal magnete permanente A in alnico V. Il filamento di tungsteno toriato, lungo 120 mm, diametro 0,35 mm, è alimentato con 50 W ed è polarizzato con 15 V positivi rispetto all'involucro della sorgente.

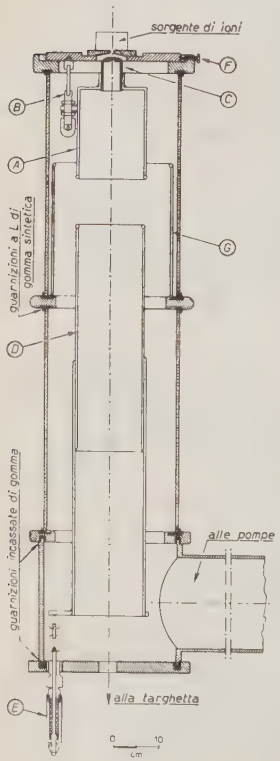

Fig. 3. – Tubo acceleratore.

Gli impulsi, rettangolari, di 600 V (misurati a vuoto) e di durata variabile di 1, o 1,5, o 5, o 10, o 15, o 50, o 100, 0 150 μs, sono applicati all'anodo cilindrico B da un generatore di impulsi avente come stadio di uscita un amplificatore catodico a bassa resistenza interna. La sorgente è percorsa da impulsi di corrente di 1,2 A e la d.d.p. ai suoi capi rimane pressochè costante per tutta la durata dell'impulso attorno ad un valore di 200 V. Il generatore di impulsi (che si trova unitamente alla sorgente di ioni nella testa del tubo) viene comandato dallo spettrografo a tempo di volo (v. sotto) il quale determina l'accensione di un diodo a gas il cui lampo di luce è raccolto da un fotomoltiplicatore a sua volta collegato con il generatore di impulsi. I comandi luminosi sono convogliati attraverso una guida di luce in plexiglas. Lo schema del generatore di impulsi e dei suoi collegamenti è dato in fig. 2.

La corrente di ioni sulla targhetta ammonta durante l'impulso a circa 3 mA.

L'afflusso di deuterio alla sorgente è regolato da un filtro di palladio avente la duplice funzione di purificatore e di valvola.

La pressione nella sorgente è di circa 10^{-3} torr. Il deuterio è rifornito con continuità da una cella elettrolitica che regola automaticamente a 1 atm la pressione a monte del palladio.

La potenza necessaria alla sorgente di ioni (riscaldamento del palladio, accensione del filamento, alimentazione del generatore di impulsi e del generatore della tensione di estrazione degli ioni) è fornita da un alternatore 160 V 1,5 kVA situato sulla testa del tubo e azionato da un motore al quale è collegato mediante un albero di bakelite.

3. – Il tubo acceleratore.

L'accelerazione e la focalizzazione degli ioni sulla targhetta sono ottenute mediante due lenti elettrostatiche. La prima di queste, che determina anche l'estrazione degli ioni dalla sorgente, è costituita da due elettrodi secondo

PIERCE [4] in modo da ottenere un fascetto conico leggermente divergente. La tensione è fornita da un generatore a radiofrequenza, montato sulla testa del tubo, ed è regolabile dal basso con un'asta di bakelite, fra 10 e 30 kV. La seconda lente, a cui viene applicata tutta la rimanente tensione acceleratrice, è del tipo a elettrodi cilindrici uguali coassiali (diametro dei cilindri

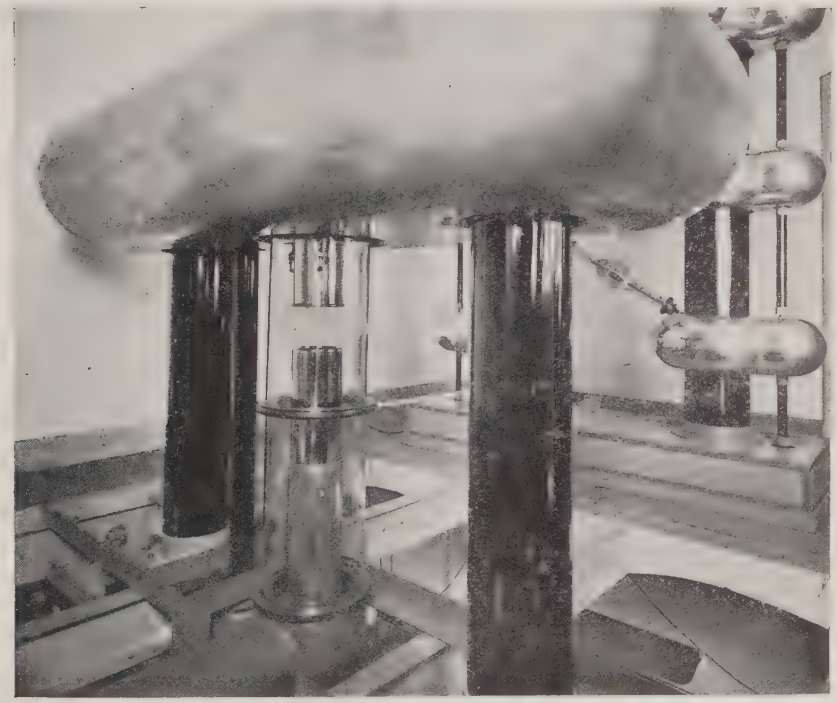

Fig. 4. – L'impianto in fase di montaggio.

150 mm, distanza fra i bordi 120 mm, regolabile). Il secondo di questi due cilindri è tenuto a -500 V per respingere gli elettroni secondari estratti dal bombardamento degli ioni nella coda e sul fondo del tubo.

L'allineamento dei vari elettrodi è ottenuto con questa successione di operazioni: 1) l'elettrodo A (fig. 3) è disposto con l'asse perpendicolare alla flangia di sostegno mediante le colonnine isolanti B; 2) il foro di uscita della sorgente è portato sull'asse dell'elettrodo estrattore C, il quale a sua volta può essere portato alla distanza voluta facendolo scorrere a cannocchiale rispetto ad A; 3) l'asse dell'elettrodo D è reso parallelo a quello di A a mezzo di tre viti calanti E mobili dall'esterno, con tenuta a soffietto; 4) gli

[4] J. R. PIERCE: *Theory and Design of Electron Beams* (New York, 1949), p. 176.

assi di A e D sono portati a coincidere spostando con le tre viti F la flangia che sostiene A.

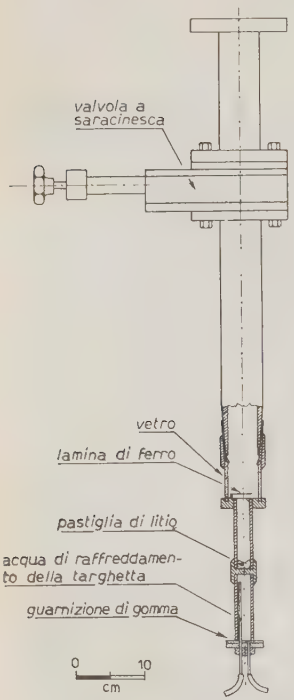

Fig. 5. – Coda del tubo.

La focalizzazione è controllata disponendo sopra la targhetta una sottile lamina di ferro, visibile dall'esterno, che diviene incandescente sotto il bombardamento degli ioni. Il diametro del fascio focalizzato è di circa 3 mm. Muovendo i sostegni E si può spostare la traccia incandescente degli ioni e centrarla esattamente sulla targhetta. In tale operazione la traccia si mantiene circolare. La focalizzazione è molto sensibile alla tensione dell'estrattore. Infatti da questa tensione dipende la leggera conicità del fascio che entra nella lente acceleratrice: variare di poco questa conicità equivale a spostare notevolmente l'oggetto dalla lente.

Uno schermo metallico cilindrico G del diametro di 250 mm e altezza 300 mm circonda la lente focalizzatrice ed è ancorato al potenziale del centro della lente. Esso definisce le condizioni al contorno del campo elettrico della lente e protegge le pareti in vetro del tubo dal bombardamento di ioni e di elettroni generati per ionizzazione del gas residuo o comunque presenti.

La fig. 4 è una fotografia dell'impianto in fase di montaggio. In essa manca lo schermo che circonda la lente.

4. – La targhetta.

La coda del tubo è intercambiabile con giunto conico.

Quella attualmente in uso (fig. 5) ha la targhetta costituita da una pastiglia di litio compressa nel fondo di un cilindro di ottone raffreddato ad acqua. Il cilindro di vetro permette l'osservazione della laminetta di ferro per il con-

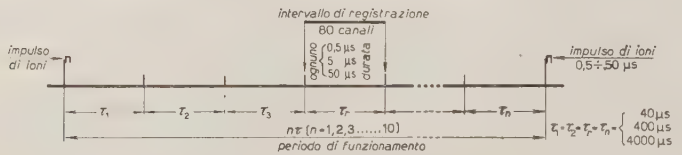

Fig. 6. – Asse dei tempi dello spettrografo a tempo di volo.

trollo della focalizzazione (la laminetta è manovrabile dall'esterno con un magnete) ed inoltre isola la coda del tubo permettendo la misura della corrente di ioni. Una grossa valvola a saracinesca consente di sostituire la coda mante-

nendo l'impianto sotto vuoto. La coda è circondata da un moderatore di paraffina per il rallentamento dei neutroni.

La produzione di neutroni è dell'ordine di $3 \cdot 10^5$ neutroni/s per μA di ioni sulla targhetta, a 350 kV (tensione normale di lavoro).

5. – Vuoto.

Il sistema di pompaggio comprende la canalizzazione di raccordo sull'alto vuoto, lunghezza cm 105, diametro cm 20; una pompa a diffusione a olio con portata di 500 l/s a 10^{-5} mm Hg; un serbatoio di vuoto di circa 90 l; e una pompa rotativa con portata di 7 l/s a pressione normale. Il vuoto preliminare è controllato con una MacLeod e l'alto vuoto con un vacuometro tipo Penning.

6. – Attrezzatura per misure di tempo di volo con neutroni.

Sono attualmente in uso due rivelatori di neutroni: uno è costituito da un insieme di 6 contatori di 40 mm di diametro riempiti con BF_3 alla pressione di 550 mm Hg, avente una superficie utile complessiva di 500 cm²: l'incertezza della profondità alla quale un neutrone è rivelato risulta pertanto di circa 2 cm; l'altro è un rivelatore a scintillazione del tipo a miscela di polveri [5]. È prevista la possibilità di disporre i rivelatori ad una distanza dalla targhetta variabile da 1 a 4 metri per effettuare misure con i neutroni col metodo del tempo di volo (vedi ad esempio [6]).

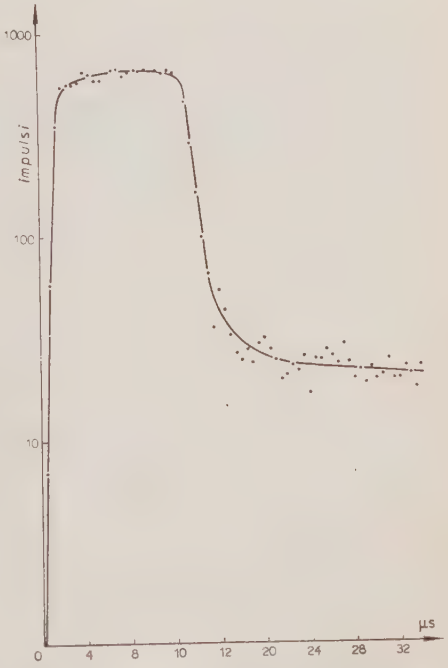

Fig. 7. – Densità dei neutroni alla superficie del moderatore di paraffina.

Un'apparecchiatura elettronica, che comanda periodicamente il generatore di impulsi della sorgente di ioni, divide il tempo tra due successive accensioni in un numero prefissato di intervalli variabile da 1 a 10, e uno a scelta di questi intervalli viene a sua volta suddiviso in 80 canali adiacenti di durata costante di 0,5, o 5, o 50 μs. Gli eventi rivelati

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[5] E. Gatti, E. Germagnoli, A. Persano e E. Zimmer: *Nuovo Cimento*, **9**, 1012 (1952).

[6] J. Rainwater e W. W. Havens: *Phys. Rev.*, **70**, 136 (1496).

nell'intervallo scelto vengono registrati su 80 corrispondenti numeratori meccanici. Tali possibilità di operazione sono illustrate nella fig. 6.

Questo spettrografo a tempo di volo è descritto in un lavoro di prossima pubblicazione su *Review of Scientific Instruments*.

Come esempio di funzionamento dell'intero impianto si riporta nella fig. 7 una curva che rappresenta la densità neutronica alla superficie del moderatore di paraffina che circonda la targhetta, in funzione del tempo. La durata dell'impulso di ioni sulla targhetta è di 10 μs. La curva è ottenuta mediante il rivelatore sottile a miscela di polveri e l'analisi temporale è effettuata mediante i canali da 0,5 μs. La curva segue per la parte iniziale l'andamento della corrente di ioni sulla targhetta, mentre la discesa lenta è dovuta alla comparativamente lunga (circa 50 μs) vita media dei neutroni nel moderatore di paraffina.

7. – Ringraziamenti.

Ringraziamo il prof. G. Bolla per il suo costante incoraggiamento e il dott. E. Germagnoli per la sua attiva collaborazione.

On the Oscillating Cylinder Viscosimeter.

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(ricevuto il 30 Novembre 1952)

Consider two vertical, coaxial cylinders between which a Newtonian liquid of density ϱ in gcm^{-3} and coefficient of viscosity η in $gcm^{-1}s^{-1}$ is contained. If, e.g.: the outer cylinder (of inner radius b in cm) is set in small oscillations at various frequencies n in s^{-1}, the liquid assumes an oscillatory motion and tends to communicate it to the inner cylinder (of radius a in cm). As the inner cylinder is suspended by a torsion wire of restoring constant r in gcm^2s^{-2}, then it will deflect through an angle θ_0, and the coefficient of viscosity η of the liquid is calculated from the following formula (1):

$$\eta = \frac{1}{2\pi}\frac{\theta_0}{\vartheta_0}\cdot\frac{b-a}{L}\frac{I\omega^2 - r}{a^3\omega}\left[1-\left(\frac{\theta_0}{\vartheta_0}\right)^2\right]^{-1/2}\exp\left[-3d/R\right]. \tag{1}$$

Where ϑ_0 is the amplitude inexorably imposed on the outer cylinder,

ω is the angular velocity of the outer cylinder,

L is the length of the inner cylinder from the surface of the liquid to the lower extremity,

also $d = (b-a)/2$,

$R = (b+a)/2$.

From the figure, we see that the experimental results of Oldroyd, Strawbridge and Toms (2) for liquid paraffin B.P. (of density 0.877 g/ml and viscosity 1.76 poises at 25 °C) agree with our theoretical equation (1) for all values of θ_0/ϑ_0. The agreement justifies the limitation in our proposed theory and the neglect of end effect.

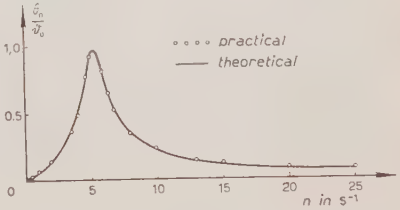

Fig. 1.

For reasons that will be pointed out, equation (1) has its advantage and disadvantage. This equation has the advantage of giving a direct relation between η

(1) The theoretical details will be published later.

(2) J. G. Oldroyd, D. J. Strawbridge, and B. A. Toms: *Proc. Phis. Soc.* B. **64**, 55 (1951).

on one side and the other constants of the apparatus on the other side; which was not fulfilled before. But, unfortunately, when theoretical comparison was made between equation (1), our previous published equation [3] and that of OLDROYD [4], it was found that this equation can be only applied for liquids of viscosity $\eta > 1$.

The authors wish to acknowledge the kind advice and encouragement given them by Prof. M. A. EL-SHERBINI during the progress of this work.

[3] A. A. K. IBRAHIM and A. M. KABIEL: *Nuovo Cimento*, **9**, 864 (1952). Here a tipographical misprint has occured: in the equation (1), the first member, (θ_0/ϑ_0), must be replaced by $(\theta_0/\vartheta_0)^2$; and in the same equation, second member, $(\gamma^2\omega^2 - D^2)$ by $(\gamma^2\omega^2 - 2D^2)$.

[4] J. G. OLDROYD: *Quart. Journ. Mech. and Applied Math.*, **4**, Pt. 3, 271 (1951).

A Note on the Raman Spectrum of Thianthrene in Solution.

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(ricevuto il 30 Novembre 1952)

Raman spectrum of thianthrene in various solutions has recently been examined by Mukerji and B. Lal who have published their observations in this Journal (1). This substance was examined in solution in carbon disulphide and in the solid state by the present author also in 1941 and it was noticed that besides the changes observed by Mukerji and Lal, the fluorescent background also showed noticeable changes with concentration. In fact, the fluorescence decreased with increasing concentration. The author is inclined to associate this phenomenon with dimerization of thianthrene under the action of light of shorter wave-lenghts. Dimerization is expected if the primary stage is activation of the molecule

$$(a) \qquad A + h\nu = A^* ,$$

followed by

$$(b) \qquad A^* + A = A_2 ,$$

A representing thianthrene and A_2 the dimeric form. As the energy of activation is not likely to be utilized by collision when concentration is small, the molecules tend to lose their energy as fluorescence. At higher concentrations the probability of collision increases and the energy of activation is utilized in the formation of a dimer. Fluorescence therefore tends to decrease with increasing concentration.

Most of the Raman lines, particularly those lying in the low frequency region, observed in solution and neither in the solid nor in the molten state, seem to be due to the dimer. It appears that there is considerable difficulty in assigning these to the formation of molecular complexes due to thianthrene molecules associating themselves with the molecules of the solvent.

(1) S. K. Mukerji and Banarsi Lal: *Nuovo Cimento*, **9**, 699 (1952).

IL NUOVO CIMENTO Vol. X, N. 1 1° Gennaio 1953

Relativistic Quantum Dynamics of a System of Interacting Particles.

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(ricevuto il 1° Dicembre 1952)

Quantum mechanics, so far, was applied either to non-relativistic systems of particles, that is systems in which the interaction is of a static type, or to systems in which the particles move in a given external field. Extension to relativistic systems of particles interacting by means of retarded or advanced forces was hindered by the fact that the corresponding variational principle (1) contains double integrals in the interaction terms besides single integrals in the terms corresponding to unperturbed motion. Such systems are of a non local type and it is impossible to describe them by a set of commuting observables corresponding to one time or, generally speaking, to one space-like surface. In fact all quantities appearing in the theory show besides an explicit dependence on the observables on a certain surface also an implicit dependence on these observables at points outside this surface. The intervention of points from the outside occurs by means of integrals of certain functions of the observables taken over the whole domain of time considered and is unavoidable in any non local theory.

It is well known from the field theory, however [1] [2], that even in non-local cases, where the traditional quantization producedures are not directly applicable, one may calculate Heisenberg's S-matrix by means of direct methods such as the method of Feynman [3], or Yang and Feldman [4].

It is the aim of the present letter to describe briefly the construction of an S-matrix for relativistic dynamics of interacting particles. Such dynamics is in general described by an action integral of the following form:

$$(1) \qquad W = -\sum_n \varkappa_n \int_{\sigma_1}^{\sigma_2} \sqrt{1-(v_i^n)^2}\, \mathrm{d}t^n + \frac{1}{2}\sum_{n\neq m}\sum e_n\, e_m \iint_{\sigma_1}^{\sigma_2} v_\mu^n\, G(q^n - q^m) v_\mu^m\, \mathrm{d}t^n\, \mathrm{d}t^m\,.$$

[1] J. Rzewuski: *Acta Phys. Polonica*, **11**, 9 (1951).
[2] J. Rayski: *Acta Phys. Polonica*, **11**, 25 (1951).
[3] R. P. Feynman: *Phys. Rev.*, **76**, 749 and 769 (1949).
[4] C. N. Yang, and D. Feldman: *Phis. Rev.*, **79**, 972 (1950).

Here Latin indices run from 1 to 3, Greek indices from 1 to 4. For both types the convention about summation over dummy indices is adopted. $v_i^n = \mathrm{d}q_i^n/\mathrm{d}t^n$ is the velocity of the n-th particle, $v_0^n = 1$. The integration limits are denoted by σ_1 and σ_2 which means that one has to integrate over t^n between points at which the line $q_i^n(t^n)$ intersects two given space like surfaces σ_1 and σ_2. We may imagine that on these surfaces some experiments are arranged and for most practical problems we may move them to $-\infty$ or $+\infty$ resp. The function G gives account of the interaction and must for relativistic reasons depend on $q^n - q^m$ by means of the invariant $(q_\mu^n - q_\mu^m)^2$. For electrodynamics $G(q^n - q^m) = \delta[(q_\mu^n - q_\mu^m)^2]$ but we shall carry out the calculations for a general G thus including into the considerations also meso-dynamics and theories with extended particles. For simplicity the term $n = m$ is excluded. It gives account of the action of one particle on itself and may be considered to be contained in the mass constant $\varkappa_n$ as a renormalisation effect (compare WHEELER and FEYNMAN [5] and FEYNMAN [6]). e_n is the charge of the n-th particles. Units are chosen in which $\hbar = c = 1$.

The equations of motion following from (1) are

$$\frac{\mathrm{d}}{\mathrm{d}t^n}\frac{\varkappa_n v_i^n}{\sqrt{1-(v_k^n)^2}} = e_n F_{i\nu}^n v_\nu^n , \tag{2}$$

where

$$F_{i\nu}^n = \frac{\partial A_\nu^n}{\partial q_i^n} - \frac{\partial A_i^n}{\partial q_\nu^n}\,; \qquad A_\nu^n(q^n) = \sum_{m\neq n} e_m \int_{\sigma_1}^{\sigma_2} \frac{1}{2}\left\{G(q^n - q^m) r_\nu^m + r_\nu^m G(q^m - q^n)\right\} \mathrm{d}t^m . \tag{3}$$

Introducing a new variable $\pi_i^n = \varkappa_n v_i^n/\sqrt{1-(v_k^n)^2}$ we may write equations (2) in a simple form

$$\frac{\mathrm{d}\pi_i^n}{\mathrm{d}t^n} = e_n F_{i\nu}^n v_\nu^n . \tag{4}$$

Integration yields

$$\begin{cases} \pi_k^n = p_k^{1n} + e_n \int_{\sigma_1}^{\sigma_2} \eta^{\mathrm{ret}}(t^n - t^{n'}) F_{k\nu}^{n'} v_\nu^{n'}\, \mathrm{d}t^{n'}, \\ \pi_k^n = p_k^{2n} + e_n \int_{\sigma_1}^{\sigma_2} \eta^{\mathrm{adv}}(t^n - t^{n'}) F_{k\nu}^{n'} v_\nu^{n'}\, \mathrm{d}t^{n'} . \end{cases} \tag{5}$$

Primes denote dependence on $t^{n'}$. p_k^{1n} and p_k^{2n} are the boundary values of π_k^n at σ_1 or σ_2 resp. They satisfy the unperturbed equations

$$\frac{\mathrm{d}p_k^{1n}}{\mathrm{d}t^n} = \frac{\mathrm{d}p_k^{2n}}{\mathrm{d}t^n} = 0. \tag{6}$$

[5] J. A. WHEELER and R. P. FEYNMAN: *Rev. Mod. Phys.*, **21**, 425 (1949).
[6] R. P. FEYNMAN: *Phys. Rev.*, **74**, 939 (1948).

In the limiting case of no interaction

$$\pi_k^n \to p_k^n . \tag{7}$$

$\eta^{\mathrm{ret}}(t)$ and $\eta^{\mathrm{adv}}(t)$ are defined by

$$\eta^{\mathrm{ret}}(t) = \begin{matrix} 1 \text{ for } t > 0 \\ 0 \quad » \quad t < 0 \end{matrix} \qquad \eta^{\mathrm{adv}}(t) = \begin{matrix} 0 \text{ for } t > 0 \\ -1 \quad » \quad t < 0 \end{matrix} . \tag{8}$$

Equations (5) may be treated by means of the Yang-Feldman method ([4]). The commutation rules for non interacting particles may be derived by application of Schwinger's method ([7]) to quantum mechanics with

$$W^0 = -\sum_n \varkappa_n \int_{\sigma_1}^{\sigma_2} \sqrt{1 - (v_i^n)^2}\, \mathrm{d}t^n , \tag{9}$$

as action integral. They are

$$[q_i^{0n}, p_k^{0m}] = i\delta^{nm}\delta_{ik} , \quad [q_i^{0n}, q_k^{0m}] = 0 , \quad [p_i^{0n}, p_k^{0m}] = 0 , \tag{10}$$

where

$$p_i^{0n} = \frac{\varkappa_n v_i^{0n}}{\sqrt{1 - (v_k^{0n})^2}} \tag{11}$$

(the superscript 0 denotes non-interacting particles). The second of the commutation rules (10) holds only if t^n and t^m correspond to the same space-like surfaces σ. For arbitrary t^n and t^m the commutation rules are derived by the usual methods but we shall not use them in this letter. The remaining commutation rules are valid generally because of the constancy of p_k^{0n}.

Now introducing the unitary operator S by the equations

$$p_k^{2n} = S^{-1} p_k^{1n} S , \tag{12}$$

combining equations (5) and (12) and using the notation

$$p_k^{0n} = \frac{1}{2}(p_k^{1n} + p_k^{2n}) , \qquad \varepsilon(t) = \eta^{\mathrm{ret}}(t) + \eta^{\mathrm{adv}}(t) = \begin{matrix} 1 \text{ for } t > 0 \\ -1 \quad » \quad t < 0 \end{matrix} \tag{13}$$

we obtain two equations

$$\pi_k^n = p_k^{0n} + e_n \int_{\sigma_1}^{\sigma_2} \frac{1}{2}\varepsilon(t^n - t^{n'}) F_{k\nu}^{n'} v_\nu^{n'}\, \mathrm{d}t^{n'} , \qquad [p_k^{0n}, S] = \frac{1}{2}\Big\{ S, e_n \int_{\sigma_1}^{\sigma_2} F_{k\nu}^n v_\nu^n \mathrm{d}t^n \Big\}, \tag{14}$$

([7]) J. SCHWINGER: *Phys. Rev.*, **82**, 914 (1951).

for π_k^n and S which may be solved by successive approximation. The first two terms in an expansion of π_k^n and S in powers of e_n are

$$\pi_k^n = p_k^{0n} + e_n \int_{\sigma_1}^{\sigma_2} \frac{1}{2}\,\varepsilon(t^n - t^{n'}) F_{k\nu}^{0n'} v_\nu^{0n'}\,\mathrm{d}t^{n'} + \dots . \tag{15}$$

$$S = 1 + \frac{i}{2}\sum_n e_n \int_{\sigma_1}^{\sigma_2} A_\nu^{0n} v_\nu^{0n}\,\mathrm{d}t^n + \dots = \tag{16}$$

$$= 1 + \frac{i}{2}\sum_{n\neq m}\sum e_n e_m v_\nu^{0n} \iint_{\sigma_1}^{\sigma_2} G(q^{0n} - q^{0m})\,\mathrm{d}t^n\,\mathrm{d}t^m v_\nu^{0m} + \dots$$

Thus the relativistic problem of interacting particles seems to be solved in principle in frames of quantum dynamics without use of the field concept. It may be regarded as an extension to quantum theory of the ideas of action at a distance advocated by WHEELER and FEYNMAN ([8]) ([5]).

Applications to practical problems shall be given in a detailed account of this work.

([8]) J. A. WHEELER and R. P. FEYNMAN: *Rev. Mod. Phys.*, **17**, 157 (1945).

IL NUOVO CIMENTO Vol. X, N. 1 1° Gennaio 1953

On the Excitation Energy of a Heavy Nucleus Described by a Real Fermi Gas.

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(ricevuto il 3 Dicembre 1952)

It is known that, according to the Thomas-Fermi model, the theoretical picture of a heavy nucleus, having mass number A, is based on the assumption that the A nucleons are independent and enclosed in a potential well of nuclear dimensions and depth 35 MeV. This model yields to previsions which are in somewhat rough quantitative agreement with experiment [1].

In order to improve the situation and to remove discrepancies between theoretical expectations and experimental data, we have attempted to deduce the equation of state of a real Fermi gas by evaluating the effect of the nuclear interactions and the dependence of the virial on the nuclear temperature T. With this aim in view, the Euler [2] and Watanabe [3] treatment, valid for the ground state of a heavy nucleus, has been extended to excited nuclei; in this connexion it may be recalled that these Authors succeeded in justifying at $T = 0$ the nuclear stability and found out that the mean kinetic energy per nucleon, calculated up to the second approximation, is larger than in the Thomas-Fermi model. Furthermore, the perturbation of the momentum distribution brought about by the latter circumstance appears to be not widely different from a Fermi distribution pertaining to an ideal Fermi gas at $T \sim 7$ MeV.

The hypotheses of these Authors, concerning the interactions between two particles, have been throughout maintained and also in the case of an excited nucleus the effect of the interactions has been accounted for by assuming that to the zero approximation the nucleons are described by plane waves.

To extend the calculations to an excited nucleus, it is essential to take properly into account the new statistical situation realized at $T \neq 0$, which can be sum-

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(1) E. Fermi: *Nuclear Physics*, (Chicago, 1950), p. 161.
(2) H. Euler: *Zeits. f. Physik*, **105**, 553 (1937).
(3) S. Watanabe: *Zeits. f. Physik*, **113**, 482 (1939); G. Puppi: *Nuovo Cimento*, **7**, 713 (1950).

marized by saying that the usual distinction between « occupied » and « unoccupied » states evolves also in that of « partially occupied » and « partially unoccupied » ones. The existence of the latter states requires a new evaluation of the transitions from an initial state defined by the impulses (p, n) into a final one of impulses (p', n'), because, of course, only transitions statistically compatible with the exclusion principle must be taken into account: in case of exchange interactions this circumstance corresponds to the exchange of all or part of the charge and spin coordinates of two interacting nucleons.

With reference to Euler symbols, be $\boldsymbol{s}$ a vector associated with the virtual motion. The condition equations $\boldsymbol{p}' = \boldsymbol{p} + \boldsymbol{s}$; $\boldsymbol{n}' = \boldsymbol{n} - \boldsymbol{s}$ establish, then, during the interaction process, the conservation of the impulse. It must, however, be emphasized that, for temperatures $T \neq 0$, they do no more imply that the virtual transitions obey to the restrictive conditions $|\boldsymbol{p}| < 1$, $|\boldsymbol{p} + \boldsymbol{s}| > 1$; $|\boldsymbol{n}| < 1$, $|\boldsymbol{n} - \boldsymbol{s}| > 1$ involving, for an unexcited nucleus, the total occupation of states within the unitary sphere of momenta.

Upon these assumptions, let us indicate with $f(p)$ the Fermi distribution function at the temperature T. The binary interactions may then be calculated, with obvious significance of the notations and following the perturbation method, by means of the matrix-element

$$(1) \qquad V_{pn}^{p'n'} = \int \mathrm{d}\boldsymbol{r}_1 \int \mathrm{d}\boldsymbol{r}_2 \, \psi_p^*(\boldsymbol{r}_1)\psi_{p'}(\boldsymbol{r}_1) J(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \psi_n^*(\boldsymbol{r}_2)\psi_n(\boldsymbol{r}_2) \,,$$

which becomes, using relative coordinates $\boldsymbol{R} = \dfrac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}$, $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$,

$$(2) \quad V_{pn}^{p'n'} = v(\boldsymbol{p}' - \boldsymbol{p}) = \frac{1}{\Omega^2} \delta(\boldsymbol{p} + \boldsymbol{n} - \boldsymbol{p}' - \boldsymbol{n}') \int \mathrm{d}\boldsymbol{R} \int \mathrm{d}\boldsymbol{r} \exp\left[\frac{2\pi i}{h} (\boldsymbol{p} - \boldsymbol{p}') \cdot \boldsymbol{r}\right] J(\boldsymbol{r}).$$

All the terms implied by ordinary and exchange interactions are obtained from relation (2). In particular, introducing polar coordinates in momentum space, the first approximation contributions to the potential energy arising from the total or, respectively, the partial exchange interactions, are:

$$\langle\, pn|J|np \,\rangle_{\text{tot. exch.}} = \frac{a}{\tau^2} \int_0^\infty J(r)\,\mathrm{d}r \int_0^\infty f(p) \sin \frac{2\pi p r}{h}\, p\,\mathrm{d}p \int_0^\infty f(n) \sin \frac{2\pi n r}{h}\, n\,\mathrm{d}n \,,$$

$$\langle\, pn|J|np \,\rangle_{\text{part. exch.}} =$$

$$= \frac{a'}{\tau^2} \int_0^\infty J(r)\,\mathrm{d}r \int_0^\infty f(p)\,[1 - f(p)] \sin \frac{2\pi p r}{h}\, p\,\mathrm{d}p \int_0^\infty f(n)[1 - f(n)] \sin \frac{2\pi n r}{h}\, n\,\mathrm{d}n \,,$$

The second approximation terms, describing the virtual motion from the initial state (p, n) to the intermediate one (p', n') and from this to the final state (n, p)

are respectively

$$-\frac{b}{\tau^2}\int_0^\infty |v(s)|\, s^2 \mathrm{d}s \int_0^\infty f(p) p^2 \,\mathrm{d}p \int_0^\infty f(n)\,|v(s+p-n)|\, n^2 \,\mathrm{d}n \cdot$$

$$\cdot \int_0^\pi \int_0^\pi \frac{F_+(s,p) F_-(s,n) \sin\vartheta_p \sin\vartheta_n}{s^2 + sp\cos\vartheta_p - sn\cos\vartheta_n} \,\mathrm{d}\vartheta_p \,\mathrm{d}\vartheta_n ,$$

$$-\frac{b'}{\tau^2}\int_0^\infty |v(s)|\, s^2 \mathrm{d}s \int_0^\infty f(p)\big[1-f(p)\big] p^2 \mathrm{d}p \int_0^\infty f(n)\big[1-f(n)\big]\,|v(s+p-n)|\, n^2 \mathrm{d}n \cdot$$

$$\cdot \int_0^\pi \int_0^\pi \frac{F_+(s,p) F_-(s,n) \sin\vartheta_p \sin\vartheta_n}{s^2 + sp\cos\vartheta_p - sn\cos\vartheta_n} \,\mathrm{d}\vartheta_p \,\mathrm{d}\vartheta_n ,$$

$$F_\pm(s,q) = 1 - f(s^2 + q^2 \pm 2sq\cos\vartheta_q) \; ; \qquad \tau = \int f(q) q^2 \mathrm{d}q \sin\vartheta_q \,\mathrm{d}\vartheta_q \,\mathrm{d}\varphi_q ,$$

where a, a' and b, b' are factors essentially expressed by proper combinations of the forces ${}^{13}A_+$, ${}^{31}A_+$, ${}^{33}A_-$, ${}^{11}A_-$ ([2]). In particular for $T = 0$, all the preceding relations convert to the Euler ones, being forbidden, for the ground state of a standard heavy nucleus, all transitions involving partial exchanges.

Let us indicate with $U^{(k)}(T)$ the total contribution at the temperature T of all the k-approximation terms. The total energy of the nucleus is then

3) $$U(T) = \sum_0^2{}_k \, U^{(k)}(T) .$$

Taking into account that the zero approximation $U^{(0)}(T)$ coincides with the result given by the Thomas-Fermi model $[U^{(0)}(T) = U_0 + bT^2;\ b = 9{,}15\ \mathrm{MeV}^{-1}]$ the excitation energy may be written in the form

(4) $$E(T) = \sum_0^2{}_k \left[U^{(k)}(T) - U^{(k)}(0) \right] = bT^2 \left[1 + \frac{4\psi_0}{\pi^2 A T^2} \sum_1^2{}_k \Delta^{(k)}(T) \right] ,$$

where $\psi_0 = 27$ MeV is the Fermi energy.

Preliminary calculations have been carried out in the region of low excitation energies assuming an interaction potential of the Gauss type. For nuclear temperatures included in the interval $0 \leqslant T \leqslant 2$ MeV the function $\Delta^{(k)}(T)$ may be expressed through the following approximate relation:

$$\sum_1^2{}_k \Delta^{(k)}(T) \simeq -\frac{A}{\psi_0} T^{9/4} .$$

The dependence of the excitation energy on the nuclear temperature is thus established by the relation

$$E(T) \simeq bT^2 \left| 1 - \frac{4}{\pi^2} T^{1/4} \right| . \tag{5}$$

and, finally, the density of the energy levels is ([4])

$$\varrho(E) \underset{\text{BETHE}}{\simeq} \varrho(E) \left| 1 - \frac{9}{2\pi^2} T^{1/4} \right|^{-1/2} \exp \left| - \frac{9A}{5\psi_0} T^{5/4} \right| . \tag{6}$$

The functions $E(T)$ and $\log \varrho(E)$ versus temperature T are plotted in Fig. 1 ($A = 100$).

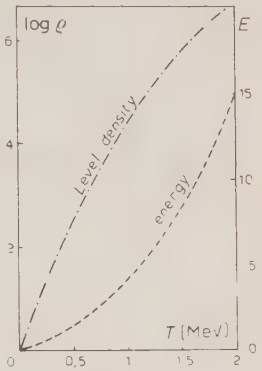

Fig. 1.

A detailed account of this work and a close discussion on the effect of different forms of interaction potentials will be soon published.

We would like to express our gratitude to Professor G. Puppi, who gave us invaluable guidance and encouragement during the course of this work. We are also grateful to Mr. B. Dequal and Mr. L. Taffara who carried out the numerical calculations.

([4]) H. A. Bethe: *Rev. of Mod. Phys.*, **9**, 81 (1937).

Ultrasonic grating remaining after stopping the supersonic waves - II.

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(ricevuto il 6 Dicembre 1952)

In a foregoing letter [1] we have noticed the first results concerning an effect observed during some investigations on colloidal solutions of starch in water subjected to supersonic waves.

We are now able to give some more details and other results. The experimental arrangement used is that of Fig. 1; in the focal plane F of a lens one ob-

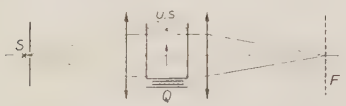

Fig. 1.

serves the diffraction spectrum produced by the quartz Q, or just the image of the luminous split, according to the presence or absence of the supersonic waves [2]. The fundamental frequency of the quartz is 0.590 MHz; we have also experimented, as we shall later relate, on the 3rd, 5th and 7th harmonics, with the respective frequencies of 1 770 MHz, 2 950 MHz and 4 130 MHz.

It is also possible to vary the power of the oscillator within a large field, raising the feeding potential of the plate from 0 to 1 200 V.

Observing the diffraction lines produced by an ultrasonic grating in liquids we noted that, for some particular colloidal solutions, the diffraction spectrum did not completely disappear with the interruption of the ultrasonic waves, as it was to be expected; that is, only the odd lines of the spectrum disappear while the even ones persist for some seconds after the opening of the circuit, fading then gradually away until they disappear. It has been possible to verify, as a fundamental fact, that this effect is intimately connected with the presence of stationary ultrasonic waves; we obtained such condition by displacing conveniently, by means of a micrometric screw, a reflecting metallic surface parallel to the piezoelectric quartz, generator of the supersonic waves.

We found that egg albumen in water or polyvinyl alcohol in methyl-, ethyl- and amyl-alcohol behave in an analogous way as starch in water (see Table I).

It is to be noticed that the couples of substances which show this effect are not real solutions; that is they are not constituted by a single phase (for instance

[1] A. Carrelli and F. Porreca: *Nuovo Cimento*, **9**, 90 (1952).

[2] R. Lucas and P. Bignard: *Journ. de Phys.*, **7**, 464 (1932).

it is known that starch is not soluble in water at room-temperature); they are suspensions of very small solid particles in a liquid, and they may be considered

Table I.

Examined Substances	Concentration g/l	Time s
Starch in water	0.1	5 ± 2
Starch in methyl alcohol	0.1	3 ± 2
Starch in benzene	various values	0
Egg albumin in water	0.2	2 ± 1
Talc in water	various values	0
Tripalmitin in methyl alcohol	» »	0
« Aldevon P » in water	» »	0
Texapon in water	» »	0
Cellulose acetate in acetone	» »	0
Glutamic acid in water	» »	0
Polyvinyl alcohol in water	» »	0
Polyvinyl alcohol in methyl alcohol	0.2	1.5 ± 0.5
Polyvinyl alcohol in ethyl alcohol	0.2	2 ± 1
Polyvinyl alcohol in amyl alcohol	0.2	3 ± 1
Colloidal silver in water	various values	0
Milk in water	» »	0
Uranium sulphate in water	» »	0

absolutely invariable during the whole lasting of our experience.

The effect is strictly dependent from the couple of substances chosen; thus starch in benzol produces no effect; the same may be said of polyvinyl alcohol in water.

From: 1st the presence of the effect connected with stationary waves, and 2nd the presence of the even lines which relies upon a direct connection with the wideness of the grating spaces — it follows that this effect has its origin in the concentration of particles suspended in the model planes.

As it has been said in our first letter, the refraction index of the solution does not differ from the one of the solvent by more than $\Delta n \simeq 10^{-6}$; this effect is therefore to be produced by an alteration of the refraction index in close proximity of the particle. If this explanation is sound, it is clear why this effect is so strictly connected with the particular nature of the solid-liquid couple we have chosen. For a given couple, in the same experimental conditions, the effect depends on the concentration of

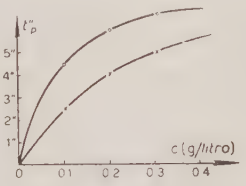

Fig. 2.

the substance in suspension: an increase of concentration increases the persistence of the lines after the interruption of the supersonic waves (Fig. 2). Therefore no proportionality exists between the persi-

stence t_p of the lines and the concentration c.

We have also noticed what follows: it is possible to give a satisfactory explanation [2] [3] [4], which is also in accord with experience, of the diffraction phenomena produced by the supersonic waves, assuming a sinusoidal variation of the index of refraction depending on the sinusoidal distribution of pression. But in such suspensions there must be an accumulation of particles in the nodal planes, and, as we have already said, around these particles there must be a variation of the index. In consequence the intensity pattern examined must differ from the theoretical previsions. We have solved this problem measuring experimentally the relative intensity of the lines diffracted by supersonic waves, in the pure solvent and in the solvent containing starch, taking care to be each time in the same experimental conditions.

We obtained the intensity measurements by taking photographs of the diffraction spectrum. With a Moll microphotometer we obtained the relative intensities of the lines in the usual way.

The obtained values are listed on Table II. It must be observed that:

1) The intensity of the diffraction lines in pure liquids (water, methyl alcohol) was in satisfying agreement with BERGMANN's [5] theoretical data, within the limits of experimental errors; hence we may assume to be in stationary conditions.

TABLE II.

Diffraction	Water–starch		Methyl alcohol–starch	
	$c = 0.1$ g/l	$c = 0.2$ g/l	$c = 0.1$ g/l	$c = 0.2$ g/l
0	100	100	100	100
1	52.0 ± 5.0	38.2 ± 4.0	65.0 ± 8.0	50.5 ± 5.0
2	25.3 ± 3.5	17.1 ± 3.0	37.5 ± 5.0	25.4 ± 3.0
3	7.7 ± 2.0	4.3 ± 1.5	13.0 ± 3.0	10.3 ± 2.0
4	4.8 ± 1.0	2.8 ± 1.0	8.5 ± 2.0	5.5 ± 1.5
5	—	1.2 ± 0.5	—	2.7 ± 1.0
6	—	1.5 ± 0.5	—	2.8 ± 1.0
7	—	0	—	0
8	—	1.3 ± 0.3	—	1.9 ± 0.5
10	—	1.0 ± 0.3	—	—

2) The values obtained show clearly the difference of intensity of the lines obtained in the two cases namely: in the pure liquid and in the liquid containing starch, also taking into account the experimental errors.

This difference depends also on concentration: to prove this, we give the values of the intensity of the lines in water-starch and methyl alcohol-starch for the two concentrations of 0.1 g/l and 0.2 g/l.

We must now find out experimentally

[3] C. V. RAMAN and N. S. NAGENDRA NATH: *Proc. of the Ind. Ac. of Sciences*, **4**, 407 (1935).

[4] C. V. RAMAN and N. S. NAGENDRA NATH: *Proc. of the Ind. Ac. of Sciences*, **1**, 79 (1936).

[5] L. BERGMANN: *Der Ultraschall* (1949), pag. 198.

the law of decrease with time of the intensity of the lines that last after the interruption of the supersonic waves. This law indeed gives us an idea of the progressive change produced by time on the particular disposition assumed by the particles within the liquid owing to the supersonic waves.

of the slit, and changing also the lasting of the supersonic waves. As the time of the passage of the light during one rotation, — 0,1 s, is not sufficient to impress the photographic plate, it is necessary, in order to obtain the photos of the lines, to reproduce at each turn exactly the same conditions.

TABLE III.

Diffraction order	Theoretical values calculated by BERGMANN	Experimental values	
		Water	Methyl alcohol
0	100	100	100
1	47.5	43.5 ± 5.0	46.0 ± 4.0
2	15.5	17.7 ± 3.5	16.5 ± 3.0
3	6.0	5.8 ± 2.0	4.5 ± 2.0
4	1.5	1.4 ± 1.0	1.3 ± 1.0

To this purpose we used a phosphoroscope which allowed us to photograph the lines lasting for some time after having switched off the oscillating circuit.

It consists of an insulating disk, placed between the monochromatic light source and the collimator slit, and turning with a uniform circular motion upon an axis parallel to the optical axis of the system. On the disk there is another slit which is brought, during the motion, to coincide with the slit of the *collimator*, allowing for an instant the passage of light.

At each turn the light passes through the slits for a time sufficiently short compared with the lasting of the lines. In our conditions for one rotation of the disk, effected in about 8 seconds, the light is allowed to pass for about 0.1 s.

On the edge of the disk there is a conducting ring of variable length, and its contact with the fixed rheofore of the supersonic oscillator, acts as a switch, triggering the beginning and the stopping of the supersonic excitation in relation with the instant of the illumination

It is therefore necessary to regulate the phosphoroscope in order that the interval, comprised between the passage of the light and the beginning of the next supersonic excitement, may be longer than the lasting of the lines in the substance we are examining.

In this way we can be sure that the

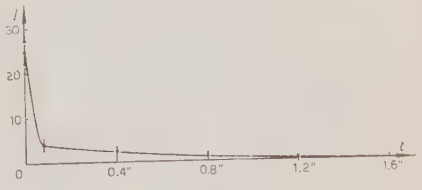

Fig. 3.

effect will happen again at each turn in the chosen conditions, and that the picture we obtain is really relative to the lines we observe after a time equal to the established interval between the opening of the circuit and the passage of the light through the slit.

In the analogous curves reported in the preceding letter ([1]) we did not take

account of such condition. Those curves gave only a qualitative pattern of the effect, therefore it is not possible to compare qualitatively the preceding curves with the ones we now show here.

In Fig. 3 we give the pattern of the intensity of the 1st order diffracted line plotted against time, assuming as origin the time of interruption of the supersonic waves. The curve has been obtained for polyvinyl alcohol in methyl alcohol at the concentration of $0.2 \pm 5 \cdot 10^{-2}$ g/l. The measurements were made at the frequency of 1 770 MHz, the supersonic waves lasting for 7 seconds.

From the pattern obtained it is easy to deduce that as soon as the supersonic waves cease, that is on the instant $t = 0$, the intensity decreases swiflty from the value it has in the spectrum obtained when the supersonic waves are excited, to the value obtained when the waves are switched off. In the following seconds the intensity I decreases much slowly and continuously.

The law of variation in time of the intensity of the supersonic waves and the lasting of the lines after the supersonic waves have ceased, must depend on three parameters: the duration, the power and the frequency of the supersonic waves.

In some following notes we shall give further details.

Drift Velocity of Electrons in BF_3.

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Laboratori CISE - Milano

(ricevuto l'11 Dicembre 1952)

As it is well known the drift velocity of electrons in boron trifluoride is of great importance for the calculation of the delay-time of neutron counters which utilise this gas.

It appeared useful to repeat the measurement of this velocity because the previous measurements have not been in agreement with each other [1] [2].

The method used is that of Allen-Rossi [1] which makes use of the measurement of the rise-time of α particles in a parallel-plate ionisation chamber. The chamber was completely built of metal to avoid the corrosion that might result from impurities in the BF_3. The plates, which had been insulated with quartz, were of a diameter of 8 cm and were 4.3 cm apart. The pulses of ionization were generated by α particles of polonium which were collimated in a direction parallel to the plates. This collimation assures a spreading of not more than 10% in the path of electrons at the pressure used. The rise time of the pulses is of order of a few microseconds. A chain of amplification had been used which gave a rise-time of 0.1 microseconds. The vacuum apparatus used for the preparation of the gas and for filling the chamber was also completely made of metal.

The gas used for the experiments was of various origin, viz: commercial BF_3 and BF_3 prepared from KBF_4.

It was impossible to use commercial BF_3 without previous purification from the relatively large amounts of impurities with high electronic affinity. The purification process consists of successive distillations in vacuum and the curve of the height of pulses in relation to the voltage applied at the chamber gives an idea of the amount of impurities still contained in the gas. Fig. 1 shows the saturation curves for commercial BF_3 and for the same BF_3 after the 1st, 2nd, 3rd and 4th distillation respectively. Further distillations did not improve the behaviour of the gas. The pressure used was 25 cm Hg.

Two kinds of chemical purification of commercial BF_3 were attempted but produced little results. In one, the gas was passed through a trap filled with dry sodium fluoride at -80 °C and in the

[1] J. Allen-Rossi: *La Report*, n. 5 (1944).
[2] J. A. Bistline: *Rev. Scient. Instruments*, **19**, 842 (1948).

other through a trap filled with concentrated sulfuric acid saturated with B_2O_3.

In the case of BF_3 prepared from KBF_4, the latter was first purified from any trace of K_2SiF_6 and other impurities through recrystallisation from ammonia water then thoroughly outgassed at 200° and successively heated to 800 °C. At this temperature, for the reaction $KBF_4 \rightleftarrows KF + BF_3$, the univariant equilibrium pressure is about 10 cm Hg [3]. The gas which was gradually liberated was condensed by means of liquid air in a reservoir. With BF_3 prepared from KBF_4 the saturation curve obtained in the chamber was identical with that of commercial BF_3 after the 4th distillation (Fig. 1). This method, which is particularly simple, may be used with advantage in order to prepare BF_3 for the counter filling.

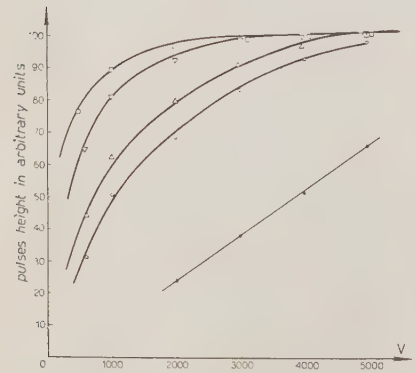

Fig. 1. – Saturation curves in BF_3 at pressure 25 cm Hg (• commercial BF_3; o 1st distillation, △ 2nd distillation; ▽ 3rd distillation; □ 4th distillation.

The drift-velocity was measured with three samples of gas and at E/P (E electric field, P pressure) varying from 1 to 5 V cm^{-1} mm^{-1} Hg. The error in the measurement was to the extent of ±15%, due partly to the dispersion in the path of electrons and partly to instrumental error when measuring the pressure and the rise-time.

Fig. 2 represents our results for comparison with those by BISTLINE and

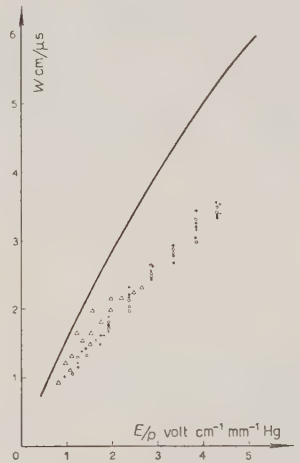

Fig. 2. – Drift-Velocity of electrons in BF_3. (• BF_3 from KBF_4; o commercial BF_3 after 2nd dist.; + commercial BF_3 after 4th dist.; △ Bistline values; — Rossi values: in this case w values are divided by 10).

ALLEN-ROSSI. The results represented agree among themselves and with those of BISTLINE.

The fact that the values of drift-velocity, in respect of boron trifluoride after being distilled twice and four times (i.e. of different purity), are equal, shows that the drift-velocity is not influenced by a small quantity of impurities contained in BF_3.

We wish to thank prof. G. BOLLA and dr. U. FACCHINI for valuable advice and discussions.

[3] J. H. DE BOER and J. A. M. VAN LIEMPT: *Rec. trav. chim.*, **46**, 124, 132 (1927).

A Search for the Production of Penetrating Secondaries by μ-Mesons Underground.

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(ricevuto il 12 Dicembre 1952)

The production of pairs of penetrating particles by cosmic rays underground has been reported by Braddick, Nash and Wolfendale [1], who explained their experimental findings assuming that μ-mesons produce penetrating secondaries on passing through matter. The cross-section for this process resulted $\sim 5 \cdot 10^{-29}$ cm²/nucleon in lead, corresponding to an interaction length of ~ 30 m; a value of the same order of magnitude had also been calculated by George and Trent [2] for the production of low multiplicity groups of penetrating particles by μ-mesons. More recently, however, Amaldi *et al.* [3] found cross-sections appreciably smaller than the ones given above, and consequently raised serious doubts about the necessity of assuming that the interaction of μ-mesons with matter is greater than the observed rates of star and shower production in photographic emulsions [4] would require.

Thus it seemed of some interest to try to re-examine, by the cloud-chamber method, the evidence for the production of penetrating secondaries by μ-mesons. A large cloud-chamber [5] was located inside a railway tunnel near Verbania (200 m a.s.l., geomagn. lat. 47° 7′ N) under ~ 55 m w.e. of granite rock (mean density 2,6 g/cm³). The chamber contained 9 lead plates 18 g/cm² thick each. Counter control was provided by a simple 3-fold coincidence telescope above the chamber, requiring a single ionizing particle able to penetrate 8 cm lead (fig. 1). The distance of tunnel ceiling from cloud-chamber top was 280 cm. By this apparatus $\sim 3\,900$ stereoscopic photographs were taken to date, 3 114 of which showed one incoming penetrating particle crossing the first 4 lead plates at least.

We will confine ourselves to consider first only the interactions of the incoming

(1) H. J. J. Braddick, W. F. Nash and A. W. Wolfendale: *Phil. Mag.*, **42**, 1277 (1951).

(2) E. P. George and P. T. Trent: *Nature*, **164**, 838 (1949); *Proc. Phys. Soc.*, A **64**, 1134 (1951).

(3) E. Amaldi, C. Castagnoli, A. Gigli and S. Sciuti: *Nuovo Cimento*, **9**, 969 (1952).

(4) E. P. George and J. Evans: *Proc. Phys. Soc.*, A **63**, 1248 (1950).

(5) A. Lovati, A. Mura, G. Tagliaferri and L. Terra: *Nuovo Cimento*, **8**, 713 (1951).

particles (assumed to be μ-mesons of $\sim 4 \cdot 10^{10}$ eV average energy) with the plates inside the chamber. In this way we minimize the possibility of misleadings deriving

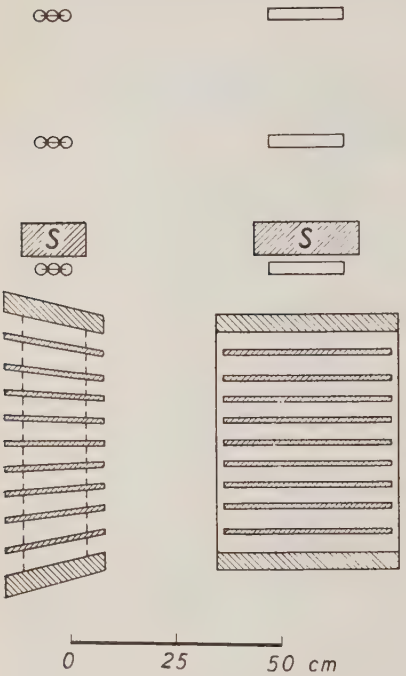

Fig. 1. – The experimental arrangement.

from random association of penetrating tracks, an occurrence that could simulate the process being searched. Further, to discriminate against trivial electron secondaries, we demand that the particle to be called a penetrating secondary should cross at least two lead plates without multiplying. Given the thickness of the plates, the required penetration brings to exclude secondaries with kinetic energy lower than about 10^8 eV, if they are either π- or μ-mesons: however, this is believed not to be a limitation, since in the process referred to by BRADDICK and coworkers the mean energy of the penetrating secondary is reported to be $\sim 10^9$ eV.

So far, we examined a total path length of 352 m lead, considering of course only the traversals of the first seven plates downwards. *No example of penetrating secondary production was found.* Even when allowing for a factor of loss (estimated, on basis of the penetrating pair angular distribution given by BRADDICK *et al.*, to be less than 50%) due to the geometry of the apparatus, our result seems to be at variance with the one of BRADDICK and coworkers, and also with some unpublished findings by GEORGE and TRENT [6] and by APPAPILLAY *et al.* [7]. It must be noticed that if we should give up the requirement of having two plates traversed by the secondary, we would have to take into consideration, in the same path length as above, two cases of production of secondary particles crossing a single plate, out of which one suffers a scattering $> 10^0$ within the plate. Here again, even regarding these events as examples of the process concerned, we would get a cross-section too low to be reconciled with BRADDICK's one.

Finally, we will refer to the events which took place above the chamber. Tracks of penetrating particles seen in the chamber were considered to be associated if clearly time coincident and reprojecting to a point within the lead S, or in the gap between S and the chamber, or in the aluminium chamber top. Applying these criteria, we found: one pair of associated penetrating particles from the aluminium, in a path length of 95 m

[6] E. P. GEORGE and P. T. TRENT, private communication.

[7] V. APPAPILLAY, A. W. MAILVAGANAM and A. W. WOLFENDALE (private communication), have obtained, by means of a cloud chamber operated at sea level under one meter lead, 6 pairs of associated penetrating particles, out of which 5 are produced close above the chamber. This gives an average length for pair production of about 60 m lead. Moreover, the chamber being equipped with a magnetic field, the mean energy of the secondary particle is found to be of the order of 900 MeV.

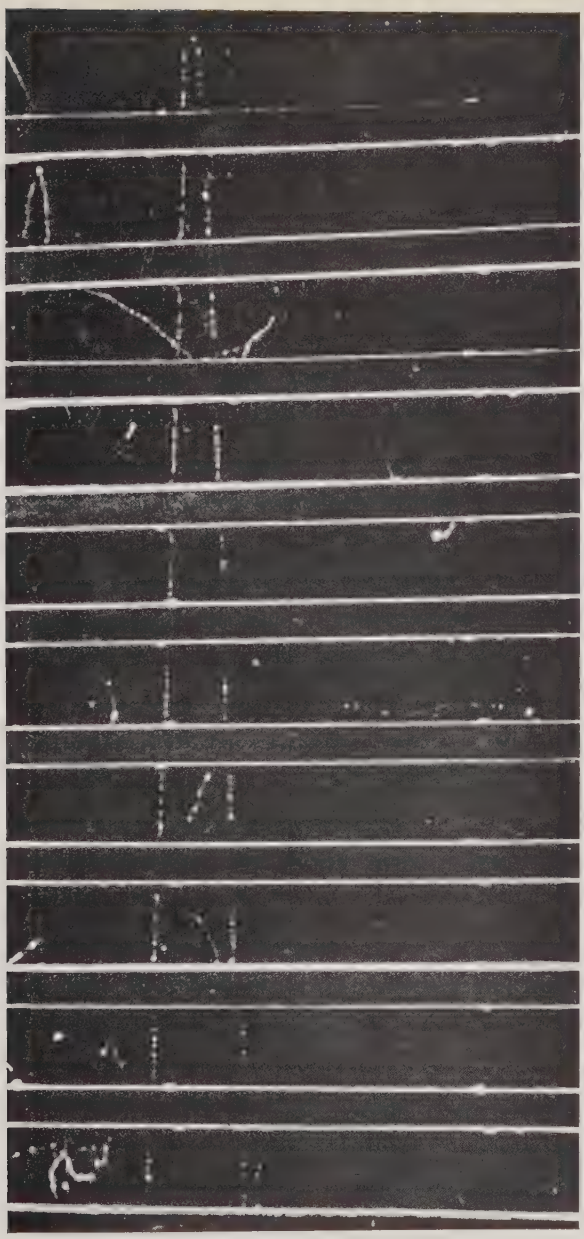

Fig. 2. – A pair of associated penetrating particles from the aluminium chamber top.

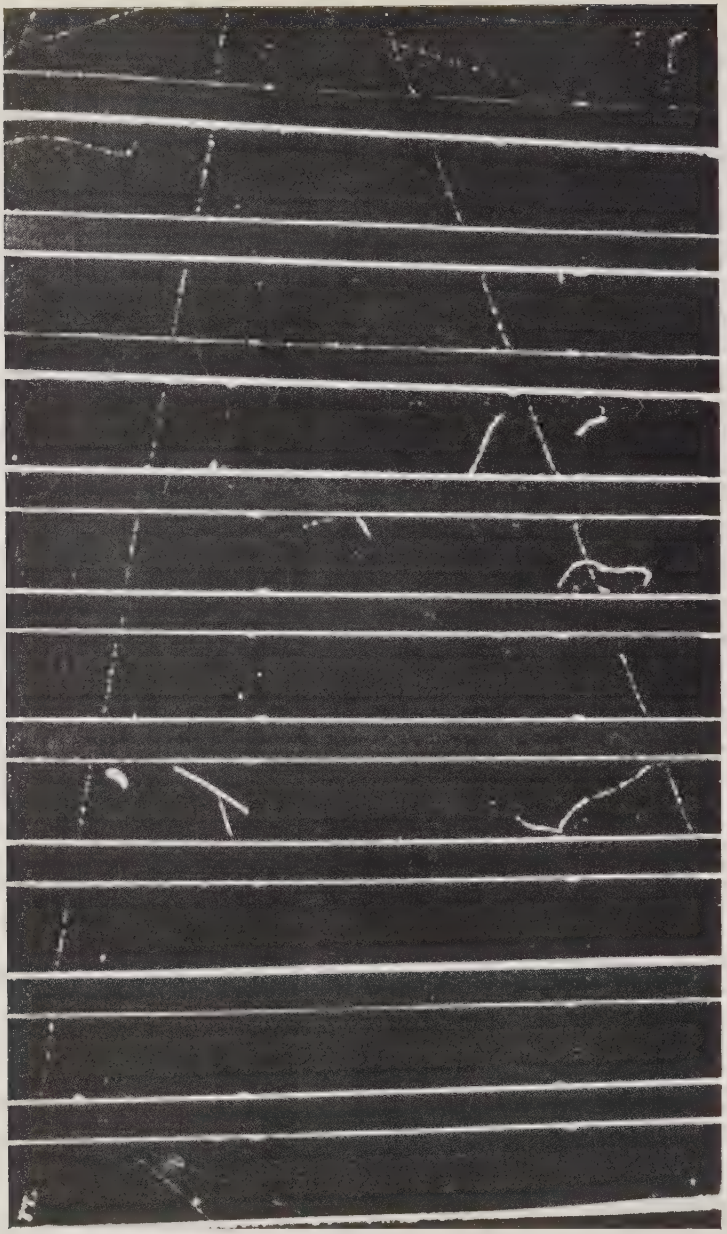

Fig. 3. – A pair of associated penetrating particles from the gap between the lead S and the chamber top.

(fig. 2); another pair from the gap (fig. 3); no pair from the lead *S*, in a path length of 249 m. It is true that in this latter instance the geometrical loss could be very important: the fact remains that once more no support was found favouring the hypothesis of fairly high μ-meson interaction rate.

It is beyond the scope of this preliminary report to discuss the possible reasons to account for the disagreement with the preceding cloud-chamber experiments. Our present conclusion is to the effect that there is no evidence of direct production of penetrating secondaries by μ-mesons at a rate greater than expected from the cross-sections of AMALDI *et al.* [3]. The research is being continued, and further results will be given on a later date.

Stelle da Protoni di Alta Energia in Emulsioni Nucleari.

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(ricevuto il 15 Dicembre 1952)

Sono state esaminate quattro lastre Ilford G5 (due da 200 μ e due da 300 μ) esposte a un fascio di protoni di 240 MeV del ciclotrone di Rochester.

L'esame, fatto per area con un ingrandimento di circa 600 diametri, è stato limitato alle zone nelle quali il fascio protonico appare piuttosto intenso.

Sono state trovate e analizzate in dettaglio 1190 stelle, con numero di rami compreso fra 2 e 8; non sono stati contati come rami nè il protone incidente, nè tracce di lunghezza inferiore a 5 μ.

La distribuzione numero di stelle in funzione del numero dei rami è la seguente:

n. di rami	2	3	4	5	6	7	8
n. di stelle	435	381	227	109	28	8	2

Il numero medio di rami per stella, considerando le stelle con due o più rami è 3,1 mentre è 3,7 considerando le stelle con tre o più rami.

(*) Attualmente presso l'Istituto Nazionale di Fisica Nucleare, Sezione di Milano.

Non sono state trovate stelle con 9 rami; alcune stelle con numero di rami maggiore di 9 non sono state prese in considerazione, perchè certamente dovute a particelle da raggi cosmici; esse hanno infatti diversi rami di notevole energia, mentre non è visibile il protone incidente.

È stato determinato lo spettro d'energia dei rami facendo distinzione solo fra protoni e particelle α; in effetti la distinzione è fra particelle ad una e a due cariche elettriche; l'approssimazione sembra però ragionevole, perchè gli effetti pick-up dovrebbero essere assai piccoli [1,2,3].

Lo spettro d'energia dei protoni, che con opportuna correzione di angolo solido, nell'intervallo 0,5 ÷ 30,5 MeV presenta un massimo fra 5,5 e 6,5 MeV, è stato determinato da misure di percorso e di densità di grani; per queste ultime, non ancora portate a termine, si sta facendo uso della relazione già suggerita

[1] J. Heidmann: *Phys. Rev.*, **80** 171 (1950).
[2] J. Hadley e H. York: *Phys. Rev.*, **80** 345 (1950).
[3] G. Bernardini *et al.*: *Phys. Rev.*, **85**, 826 (1952).

da RITSON [4]

$$N = N_0(1 - \exp[-I/I_0]),$$

dove $N_0 = 150$ grani/50 μ rappresenta la densità di saturazione dei grani, I è la ionizzazione specifica del protone e I_0 un parametro che dipende dalla sensibilità e dallo sviluppo dell'emulsione; le lastre sono state preventivamente tarate con i protoni da 240 MeV.

In molti casi per distinguere fra protoni e particelle α sono state fatte misure di « gap ».

Lo spettro d'energia delle particelle α, che ha il massimo fra 10,5 e 12,5 MeV, è stato determinato da sole misure di percorso.

Nello spettro d'energia sia dei protoni che delle particelle α, è stato notato che con l'aumentare del numero dei rami, il massimo si sposta verso le energie più basse, il che è probabilmente consistente col fatto che con l'aumenatre del numero dei rami delle stelle, aumenta il numero di rami di energia via via più piccola.

La distribuzione angolare dei rami è stata determinata misurando l'angolo formato fra la proiezione dei rami sul piano della lastra e la direzione del fascio protonico incidente.

La maggior parte dei rami veloci ($E > 30$ MeV) sono proiettati in avanti rispetto al verso dei protoni incidenti, entro un angolo di circa 120° sul piano della lastra; essi sono tanto più collimati quanto maggiore è la loro energia.

La distribuzione angolare risula anisotropa anche per i rami neri ($E < 30$ MeV) contrariamente a quanto sarebbe ragionevole attendersi se essi fossero tutti dovuti a un processo di pura evaporazione [5]; il contributo isotropo è di circa 2/3 come risulta dalla fig. 1. Come già è stato osservato da BERNARDINI *et al.* [6] questo

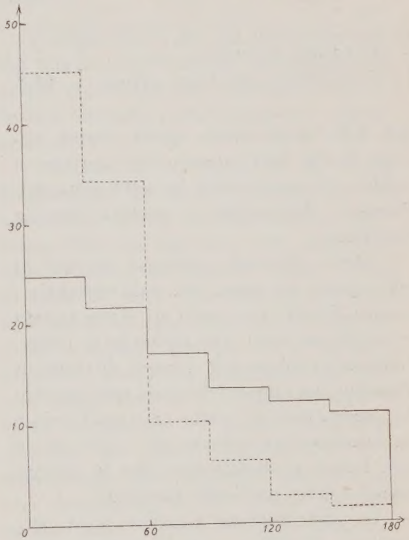

Fig. 1.

starebbe a significare che l'attribuzione di tutti i rami neri a un processo evaporativo non è probabilmente del tutto esatta.

Il numero totale di stelle e il numero di stelle con rami veloci in funzione del numero dei rami neri è dato nella tab. I; nella tab. II è data la percentuale di

TABELLA I.

N. di rami neri	0	1	2	3	4	5	6	7	8
N. di stelle	42	319	315	308	125	51	21	7	2
N. di stelle con rami veloci	42	319	162	154	33	7	1	0	0

[4] H. FISHMANN e A. M. PERRY: *Phys. Rev.*, **86**, 167 (1952).

[5] K. J. LE COUTEUR: *Proc. Phys. Soc.*, A **63**, 259 (1950).

[6] A. M. PERRY: *Phys. Rev.* **86**, 497 (1952).

stelle con rami veloci in funzione del numero totale dei rami.

Fra le 1 190 stelle esaminate il 39,6% non ha rami veloci; ha un solo ramo veloce il 46,2% e due rami veloci il 14,2%.

La somma delle energie dei rami visibili delle stelle risulta quasi sempre minore di 120 MeV; questo significa che in media, per ogni stella, la metà circa dell'energia disponibile è portata via da neutroni.

Questi risultati, sebbene relativi al solo esame per area, non sono sostanzialmente diversi da quelli di BERNARDINI *et al.* [3] per quel che riguarda la distribuzione angolare e il numero di stelle in funzione del numero dei rami neri, mentre la percentuale di stelle con rami veloci in funzione del numero dei rami neri è più bassa, probabilmente per la minore energia delle particelle incidenti.

La distribuzione angolare e la percentuale di stelle con rami veloci in funzione del numero totale dei rami, sono in buon accordo con i risultati di FISHMANN e PERRY [4].

Si sta ora facendo l'esame per traccia protonica; in 30,06 metri sono state trovate 62 stelle, 40 scattering di 4° o più e uno stopping; il corrispondente libero cammino medio è di circa 29,2 cm per tutti gli eventi osservati.

TABELLA II.

N. totale di rami	2	3	4	5	6	7	8
N. di stelle con rami veloci	64,8	59,5	59,4	59,6	28,5	12,5	0

Tenendo presente che per il calcolo del libero cammino medio anelastico il numero degli scattering va opportunamente ridotto, il fenomeno della trasparenza nucleare per i protoni da 240 MeV, come già trovato da PERRY [6] appare evidente; infatti il libero cammino medio per le emulsioni G5 corrispondente alla sezione d'urto nucleare geometrica è di circa 25 cm. Maggiori dettagli saranno dati più tardi.

Desidero esprimere i miei più vivi ringraziamenti al prof. R. E. MARSHAK per avermi messo a disposizione le lastre e per i suggerimenti datimi all'atto di iniziare il lavoro e al dr. D. M. RITSON per gli utili consigli. Sono grato anche ai proff. G. SALVINI e G. FRONGIA per alcune proficue discussioni.

LIBRI RICEVUTI E RECENSIONI

A. GATTERER – *Grating spectrum of iron.* (Spettro di reticolo del ferro). Specola Vaticana, Città del Vaticano, 1951. 47 tavole fotografiche formato 24 × 30. Testo di 16 pagine.

A sedici anni di distanza dalla pubblicazione dei due atlanti contenenti lo spettro di prisma del Ferro (arco e scintilla) ad opera dei P.P. GATTERER e JUNKES, vede ora la luce un nuovo atlante dello spettro del Ferro derivato da spettri presi con reticolo concavo. Scopo principale di questi atlanti è fornire uno spettro di riferimento a chi esegue delle analisi spettrochimiche. Come tali essi devono pertanto piegarsi alle esigenze della tecnica. Infatti, mentre per molti anni il lavoro di analisi spettrochimica è stato fatto impiegando spettrografi a prisma, dopo la guerra, invece, soprattutto per l'influenza esercitata dall'industria degli Stati Uniti, sono entrati nell'uso corrente molti spettrografi a reticolo concavo che hanno il vantaggio di dare una dispersione praticamente costante in tutto lo spettro e di prestarsi altrettanto bene per il visibile, l'ultravioletto o l'infrarosso fotografico. Questo nuovo orientamento ha fatto sì che è stata subito sentita la mancanza di adeguate tavole spettroscopiche delle sorgenti di riferimento più usate: l'arco e la scintilla del Ferro. Spettroscopisti ed astrofisici hanno dovuto far ricorso alle vecchie tavole dell'atlante di KAYSER e RUNGE del 1881 oppure adattarsi ad impiegare gli atlanti degli spettri prismatici con grave disagio e notevole fatica.

Giunge quindi più che opportuna questa nuova pubblicazione dell'infaticabile padre GATTERER per colmare la grave lacuna esistente nella letteratura spettroscopica.

L'atlante è costituito da 47 tavole fotografiche, del formato 24 × 30 cm, ciascuna delle quali contiene due pose graduate dell'arco del ferro, due della scintilla ed infine uno spettro d'arco ed uno di scintilla messi a confronto. Le lastre originali sono state prese con uno spettrografo a reticolo concavo di 21 piedi di raggio in montatura stigmatica che dava una dispersione di 5 Å/mm costante in tutto lo spettro; le tavole sono state ricavate da queste ingrandendo nello stesso modo tutti gli spettri fino ad avere una dispersione di 0,7 Å/mm. Su ogni spettro sono segnate le righe principali e la λ è data, quando possibile, con l'approssimazione di 0,001 Å. Una novità importante è l'introduzione dell'arco di Zr come sorgente di riferimento nella zona spettrale fra 5 300 e 8 900 Å. In questa parte dello spettro la scintilla del Fe è poverissima di righe ed anche l'arco del Fe non ne è molto ricco; l'arco del Zr si presta invece molto bene sia per la facilità di produzione dello spettro, sia perchè possiede righe sufficientemente intense e ben ripartite in lunghezza d'onda. Altro grande vantaggio di questa sorgente è quello di avere delle bande di ZrO caratteristiche e ben definite cosicchè è facile individuare la zona spettrale corrispondente. Esse inoltre hanno una grande importanza astrofisica essendo presenti in molte stelle di tipo avanzato.

Benchè dedicato principalmente agli spettrochimici, questo atlante non di-

mentica gli astrofisici; infatti le tavole 46 e 47 sono dedicate proprio agli astronomi spettroscopisti, perchè contengono tutte le righe del Ne fra 5900 e 8900 Å che spesso vengono usate per identificare le righe rosse ed infrarosse negli spettri stellari.

Non è necessario dire che gli spettri sono quanto di meglio si possa avere con i moderni strumenti; il nome dell'Autore è di per sè garanzia sufficiente. Il testo che accompagna l'atlante dà tutte le condizioni sperimentali: strumento, ottica usata, condizioni di eccitazione degli spettri, ecc.

La purezza degli elettrodi usati è elevatissima, ne segue che solo un piccolo numero di righe spurie deboli si trovano nelle tavole dell'atlante. Le lunghezze d'onda delle righe sono state accuratamente vagliate; esse provengono in massima parte da fonti autorevoli, ma un notevole numero di esse sono state determinate dell'Autore direttamente.

La presentazione dell'atlante è nel medesimo stile delle altre pubblicazioni uscite dallo stesso laboratorio e quindi eccellente; la stampa del testo nitida ed elegante e le tavole degli spettri chiarissime e di facile comprensione.

Siamo certi che spettroscopisti ed astrofisici di tutto il mondo saranno oltremodo grati al Padre Gatterer per questa sua nuova fatica.

G. Righini

H. Brückner – *Gastafeln.* Un vol. di 222 pagg. con 15 figure. Verlag von R. Oldenbourg. München 1952.

È una raccolta di dati relativi alle proprietà fisiche e ad altre proprietà tecnicamente importanti dei gas combustibili e di altri gas usati industrialmente. Per ciò che riguarda le proprietà fisiche generali di questi gas, non vi è detto niente di più di quanto già ordinariamente si trova in altri manuali e raccolte del genere. Il libro potrà riuscire utile ai tecnici, perchè nell'ultima parte vengono diffusamente illustrate, anche con esempi numerici, tabelle di dati riguardanti proprietà tecnologicamente importanti, quali i limiti di infiammabilità, la temperatura e la velocità di propagazione delle fiamme, ed altre proprietà di molti combustibili liquidi e gassosi. Mancano quasi completamente i riferimenti bibliografici.

G. Careri

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